Recursive Preferences, the Value of Life, and Household Finance*

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Abstract

We analyze lifecycle saving strategies with recursive utility calibrated to match empirical estimates for the value of a statistical life. We show analytically that, with a positive value of life, risk aversion amplifies the impact of uncertain survival on the discount rate, and thereby reduces savings. This "carpe diem" consumption motive counteracts the well-known precautionary savings motive that arises from income risk. In a calibrated lifecycle model with multiple risks, we find that the "carpe diem" motive dominates, so that lifecycle savings decrease when risk aversion rises. Our model also predicts that risk aversion lowers stock market participation. Our findings stand in contrast to studies that implicitly assume a negative value of life.

Keywords: recursive utility, lifecycle model, value of life, risk aversion, saving choices, portfolio choices.

JEL codes: D91, G11, J17.

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1 Introduction

Household finance and the economic appraisal of the value of life are central issues in the economics of aging. Although both topics make use of similar theoretical foundations based on micro-economic lifecycle models, they are developed in separate strands of the economic literature with apparently little exchange between them.

On the one hand, the household finance literature tackles questions related to optimal lifecycle consumption-saving and financial portfolio choices. These questions are of first-order importance for analyzing and designing efficient pension systems, and more generally for thinking about saving incentives. To answer such questions, many sophisticated techniques have been adopted, such as recursive models representing preferences à la Kreps and Porteus (1978). These preferences enable the disentangling of risk aversion from intertemporal elasticity of substitution (IES, henceforth), which has been shown to help explain lifecycle consumption and risky saving patterns. From a modeling standpoint, most of this literature assumes that individuals die with exogenous mortality rates. Mortality being exogenous, there is no explicit trade-off between consumption and life duration. The notion of the value of life, which measures the individual willingness to give up consumption for living longer, is generally not considered.

On the other hand, the literature on the value of life is centered around questions related to mortality risk reduction. Public policies aiming at lowering mortality risk—such as road safety investments or public health campaigns—are typically quite expensive and evaluated through cost-benefit analyses. This requires comparing the monetary amounts for financing the new policy to human lives that have been saved. Therefore, having an estimate of the value of life is key, and a large share of the literature has focused on obtaining robust empirical evidence. However, little use has been made of the theoretical advances that were derived in household finance, and especially of the recursive models that allow for disentangling risk aversion from the IES.[1]

[1] Exceptions include Hugonnier, Pelgrin, and St-Amour (2013) and Córdoba and Ripoll (2017), which we further discuss in Section 6.3.

In this paper, we investigate the benefits of importing knowledge from the value of life literature into household finance. In particular, we show that a proper calibration of the value of life brings new insights on the determinants of savings and portfolio choice. We motivate our work by a basic two-period model, which is sufficient to show that the value of life may significantly affect time discounting and agents' decisions, even though mortality is exogenous. The main contribution of the paper consists in the development and analysis of a large-scale quantitative lifecycle model that is standard in most aspects but adds an additional parameter offering the flexibility to match empirical estimates of the value of life.

When the model is calibrated to match the empirical estimates of the value of life, the conclusions differ from those derived in the literature with models that implicitly assumed a negative value of life. In particular, in contrast to the seminal contributions of Gomes and Michaelides (2005, 2008), our model predicts that more risk averse agents tend to save less and to participate less in the stock market. The reason is that, with non-additive preferences and a positive value of life, risk aversion amplifies the role of survival risk in the discount rate. The resulting higher discount rate reduces the agent's propensity to save. One can interpret this as a "carpe diem" consumption motive, which induces more risk-averse agents to consume more today because they could be dead tomorrow. Quantitatively, this "carpe diem" motive turns out to dominate the standard precautionary motive. Our result that more risk averse agents participate less in the stock market and opt for more conservative portfolios re-aligns the model prediction with the common understanding of risk-taking behavior as well as with empirical evidence (see, e.g., Dohmen, et al. 2011).

The rest of the paper is organized as follows. In Section 2, we introduce the concept of the value of a statistical life and, with the help of a basic two-period model, explain why it is important for household finance. We then present our quantitative lifecycle model in Section 3. We specify utility functions in Section 4 and describe the calibration in Section 5. Section 6 presents the results and discusses how they relate to the previous literature. We conclude in Section 7.

2 The value of a statistical life

2.1 A marginal rate of substitution between survival probability and consumption

We present the definition for the value of a statistical life (VSL, henceforth), as well as the rationale behind it. As a starting point for this definition we need the concept of lifetime utility. In standard lifecycle models with uncertain life duration, the lifetime utility of an agent depends on consumption levels at different ages and survival probabilities, as well as possibly on other factors, such as health or leisure, for instance. Denoting by c_t the consumption at age t, by p_t the survival probability between ages t and $t+1$, and by x_t the vector gathering other factors, the lifetime utility of an agent can be expressed as $U(c_0, \ldots, c_t, \ldots, p_0, \ldots, p_t, \ldots, x_0, \ldots, x_t, \ldots)$. The functional form of the function U is generally further specified, for instance through expected utility models. However, the concept of VSL is probably easier to understand if we abstract from specific functional forms. The VSL can be defined as a marginal rate of substitution between survival probability and consumption. Formally, the VSL at date t, denoted VSL_t, is defined as

$$VSL_t = \frac{\frac{\partial U}{\partial p_t}}{\frac{\partial U}{\partial c_t}}. \tag{1}$$

It is worth explaining why the marginal rate of substitution between survival probability and consumption is called "Value of a Statistical Life". By definition of the marginal rate of substitution, an agent would be willing to exchange εVSL_t units of period-t consumption for an increase of ε in her survival probability (where $\varepsilon > 0$ is infinitesimally small). Considering a population of $\frac{1}{\varepsilon}$ identical agents, they would in aggregate be willing to pay $\frac{1}{\varepsilon} \times \varepsilon VSL_t = VSL_t$ to increase the expected number of survivors by $\frac{1}{\varepsilon} \times \varepsilon = 1$. In other words, VSL_t represents the aggregate willingness-to-pay to save (on average) one life. This explains why the terminology of the value of a statistical life has been coined.

2.2 Empirical literature on the VSL

As mentioned in the introduction, the VSL is a key parameter for cost-benefit analysis in policy designs. For example, about 85% of the benefits of the Clean Air Act are related to mortality risk reduction, as computed by the US Environmental Protection Agency (see their 2011 prospective report). In other words, using VSL estimates that would be off by a factor of 2 would also lead to under- or overestimate the benefits of the Clean Air Act by a factor of about 1.8. This illustrates the importance of having good estimates of the VSL.

Many researchers (e.g., see the review in Viscusi, 2003) and institutions (e.g., Environmental Protection Agency and Food and Drug Administration in the US) put significant effort into obtaining estimates from observed behaviors. Essentially, one wants to find empirical evidence on the willingness-to-pay for mortality risk reduction. One approach is too look at wage-risk trade-offs. Another is to look at the willingness-to-pay to get safer cars, safer homes, etc. Direct questionnaires can also be informative.

Although empirical estimation proves difficult, there is a wide consensus that for most people the value of life is positive and large. There is, of course, some heterogeneity across studies and social context, but for a country like the US, a VSL of 6 to 7 million dollars is considered a reasonable estimate.

2.3 Why should we care for the VSL even if mortality is exogenous?

In this section we want to emphasize that even if mortality is exogenous, assumptions made regarding the value of life have major consequences on savings behavior. In order to make this point we focus on a simple setting where agents live at most two periods (0 and 1), and only care for consumption and survival.[2] Their utility is therefore a function $U(c_0, c_1, p_0)$ using the notation of Section 2.1. A popular way to further specify this utility function is to use a recursive approach. This approach requires to first compute the agent's utility in period 1 (depending on her

[2]Most of the simplifying assumptions in this subsection will be relaxed in the large-scale quantitative model of Section 3.1.

consumption and on whether she is alive or not), and then to derive the period-0 utility using a recursive expression à la Kreps and Porteus (1978). Period-1 utility, which depends on consumption c_1 and on survival status $\xi_1 \in \{\text{alive, dead}\}$, is denoted by $U_1(c_1, \xi_1)$. The period-0 utility can then be expressed as

$$U(c_0, c_1, p_0) = u(c_0) + \beta\phi^{-1} E_{p_0}[\phi(U_1(c_{1,}\xi_1))], \tag{2}$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is an instantaneous felicity function, $\beta \in (0, 1)$ is a discount factor, and $\phi : Im(U) \to \mathbb{R}$ is an increasing and concave function governing risk aversion. We assume that the instantaneous felicity functions when alive are the same at dates 0 and 1, so that $U_1(c_1, \text{alive}) = u(c_1)$. The agent has no bequest motive and does not care about her consumption after death. Consequently, $U_1(c_1, \text{dead})$ is independent of c_1 and we set $U_1(c_1, \text{dead}) = u_d \in [-\infty, +\infty]$, since infinite evaluations are theoretically possible. The value of u_d governs the felicity gap between life and death. Lifetime utility is given by

$$U(c_0, c_1, p_0) = u(c_0) + \beta\phi^{-1} \left(p_0\phi(u(c_1)) + (1 - p_0)\phi(u_d)\right). \tag{3}$$

The value of life. Applying definition (1) we get

$$VSL_0 = \frac{\beta \left(\phi(u(c_1)) - \phi(u_d)\right)}{u'(c_0)\phi' \left(\phi^{-1} \left(p_0\phi(u(c_1)) + (1 - p_0)\phi(u_d)\right)\right)}. \tag{4}$$

The VSL is therefore connected to the value of u_d, being positive if $u_d < u(c_1)$ and negative if $u_d > u(c_1)$.

It is standard in the household finance literature to pick values of u_d that provide tractable specifications without paying attention to the implied value of life. A commonly used specification is the Epstein-Zin utility function, given by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\phi(u) = \frac{((1-\sigma)u)^{\frac{1-\gamma}{1-\sigma}}}{1-\gamma}$, where $\frac{1}{\sigma}$ is the IES and γ controls risk aversion. Typically both σ and γ are assumed to be larger than 1. Maximal tractability is then obtained by setting $u_d = 0$ so that $\phi(u_d) = 0$. Indeed, in such a case equation (3) provides

$$U(c_0, c_1, p_0) = \frac{c_0^{1-\sigma}}{1 - \sigma} + \frac{\beta}{1 - \sigma} p_0^{\frac{1-\sigma}{1-\gamma}} c_1^{1-\sigma}, \tag{5}$$

which is homothetic, a very helpful property for deriving solutions of utility maximization problems. This convenient specification, however, assumes that the death felicity u_d is greater that the alive felicity, no matter the consumption level: $u_d > u(c)$ for all c. This implicitly corresponds to a negative value of life, because γ is assumed to be greater than one.[3]

The value of life and time discounting. Even with exogenous mortality, using a good estimate for the VSL is important. Let us, for example, consider the implications on the consumption discount rate between periods 0 and 1. This discount rate can be defined as

$$\delta_0 = \left.\frac{\frac{\partial U}{\partial c_0}}{\frac{\partial U}{\partial c_1}}\right|_{c_0=c_1} - 1, \tag{6}$$

which is the rate of change of marginal utility for a constant consumption stream. With preferences represented by (3), we obtain the following expression:

$$\delta_0 = \frac{1}{\beta p_0}\frac{\phi'(\tilde{u})}{\phi'(u(c_1))} - 1, \tag{7}$$

where

$$\tilde{u} = \phi^{-1}\left(p_0\phi(u(c_1)) + (1-p_0)\phi(u_d)\right) \tag{8}$$

is the certainty equivalent (in felicity levels) of a lottery that gives felicity $u(c_1)$ with probability p_0 and u_d with probability $1-p_0$. In the case where ϕ is affine, ϕ' is constant, and equation (7) reduces to $\delta_0 = \frac{1}{\beta p_0} - 1$. For affine ϕ, therefore, mortality contributes to impatience in a very simple way, and in particular independently of the constant u_d. This however does not extend to non-additive models with strictly concave function ϕ. To see the impact of ϕ, rewrite equation (7) as

$$\delta_0 = \frac{1}{\beta p_0}\exp\left(\int_{\tilde{u}}^{u(c_1)} \lambda_\phi(u)du\right) - 1, \tag{9}$$

[3] One can check that $\frac{\partial U(c_0,c_1,p_0)}{\partial p_0} = \frac{\beta}{1-\gamma}p_0^{\frac{\gamma-\sigma}{1-\gamma}}c_1^{1-\sigma} < 0$, no matter p_0 and c_1. Note, moreover, that preferences represented as in (5) may also be represented by $V(c_0,c_1,p_0) = \left(c_0^{1-\sigma} + \beta p_0^{\frac{1-\sigma}{1-\gamma}}c_1^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, which is related to (5) by $V(c_0,c_1,p_0) = ((1-\sigma)U(c_0,c_1,p_0))^{\frac{1}{1-\sigma}}$. The representation V is often used, because it provides a positive utility function even when $\sigma > 1$. However, when $\sigma > 1$, the representation V implicitly assumes that the utility of death is $+\infty$, implying a negative value of life, just like when U is used (see also Section 6.3).

where $\lambda_\phi(u) = -\frac{\phi''(u)}{\phi'(u)}$ is a measure of the concavity of ϕ and thus a measure of risk aversion. When ϕ is concave, the factor $\frac{1}{\beta p_0}$ is multiplied by the quantity $\exp\left(\int_{\tilde{u}}^{u(c_1)} \lambda_\phi(u)du\right)$, which intertwines risk aversion, mortality, and impatience. This exponential term depends on $\tilde{u}$, defined in equation (8), and therefore on how u_d compares to $u(c_1)$.

Consider first the case where $u_d < u(c_1)$, which means a positive value of life. Then, the greater the concavity of ϕ and the higher the mortality rate $1 - p_0$, the larger the expression $\exp\left(\int_{\tilde{u}}^{u(c_1)} \lambda_\phi(u)du\right)$ and the discount rate δ_0. In other words, if the value of life is positive, risk aversion amplifies the impact of mortality on the discount rate: in the presence of mortality risk, more risk averse agents are more impatient. One can interpret this "carpe diem" consumption motive as follows: the agent does not want to take the risk of having both a short life and a low average consumption.

It is worth contrasting this case with the one where $u_d > u(c_1)$, implying a negative value of life. We now have $\tilde{u} > u(c_1)$, so that the factor $\exp\left(\int_{\tilde{u}}^{u(c_1)} \lambda_\phi(u)du\right)$ is smaller than one, thus reducing—instead of amplifying—the impact of mortality on the discount rate .

Overall, for given IES and risk aversion (i.e., given functions u and ϕ, respectively), a model with a negative value of life will predict a lower rate of discount than the same model with a positive value of life. As a lower discount rate means larger savings, we find that underestimating the value of life (and in particular using a negative value) is likely to exaggerate the propensity to save. This may in turn generate biased predictions for portfolio choice. However, the question remains how important the "carpe diem" consumption motive is compared to the precautionary savings motive. To answer this, we turn to a quantitative analysis in the next sections.

3 A quantitative lifecycle model

Having explained why it is important to carefully calibrate the value of life, we present in this section a lifecycle model that is mostly standard, but in which the felicity level of the death state (u_d in the previous section) will be fixed to fit empirical estimates of the VSL.

3.1 The setup

We consider a partial equilibrium economy populated by an agent endowed with recursive preferences and facing several risks: a mortality risk, an income risk, and an investment risk through risky financial returns. The agent may save through a bond and a risky asset (similar to a stock). Time is discrete, and the agent's age is denoted by t. The agent enters the model at working age, $t = 0$. There is a single consumption good, whose price serves as a numeraire.

Mortality risk. The agent faces mortality risk, which is assumed to be exogenous and independent of any other risk in the economy. If alive at date t, the agent survives to date $t+1$ with probability p_t. There exists a date T_M, such that the probability to live after T_M is $p_{T_M} = 0$.

Labor income risk. At any age, when alive, the agent receives an income denoted y_t. The agent exogenously retires at age T_R. During retirement ($t \geq T_R$), the agent receives a constant pension income $y_t = y^R$. During working age $t < T_R$, the agent earns a risky labor income $y_t = y_t^L$, which is subject to both persistent shocks, π_t, and transitory shocks, ϑ_t:

$$y_t^L = \bar{y} \exp\left(\mu_t + \pi_t + \vartheta_t\right), \tag{10}$$

$$\pi_t = \rho \pi_{t-1} + \upsilon_t. \tag{11}$$

The two independent processes $(\upsilon_t)_{t \geq 0}$ and $(\vartheta_t)_{\geq 0}$ are IID and normally distributed with mean 0 and respective variance σ_υ^2 and σ_ϑ^2. The quantity $\bar{y}$ in (10) represents a constant wage rate, while $(\mu_t)_{t \geq 0}$ is a deterministic process that contributes to fit the wage process to the data and in particular the hump-shaped pattern of income during active age. The parameter ρ in (11) drives the persistence of the process π.

Financial risk and security markets. The agent has the opportunity to save through a risk-free one period asset (similar to a T-Bill) and a risky asset (similar to a stock). The bond pays in the subsequent period a risk-free gross return, R^f, which is constant and exogenous. The risky return is:

$$\ln R_t^s = \ln(R^f + \omega) + \nu_t,$$

where ω represents the average risk premium of stocks over bonds, while the financial risk $(\nu_t)_{t\geq 0}$ is an IID normally distributed process with mean 0 and variance σ_ν^2. The financial shocks, $(\nu_t)_{t\geq 0}$, are potentially correlated with the persistent income shocks, $(\upsilon_t)_{t\geq 0}$. The correlation is assumed to be constant and is denoted κ_υ.

The agent must pay a cost $F \geq 0$ to participate in the stock market, which may be interpreted as an opportunity cost to discover how the stock market works. In our baseline model, we assume it is a flat once-in-a-lifetime cost: if the cost is paid at a given date t, the agent can freely trade stocks at date t and at any date afterwards. Section 6.2 examines alternative cost specifications found in the literature.

Timing and notation. At the beginning of every period, the agent first learns the realizations of financial and labor income shocks and whether she is alive or not. She thus knows the amount of her current savings and, if she is alive, her current income. More precisely, at any date t, we assume that the agent knows the entire history of all shocks up to date t, which is formalized by the natural filtration $(\mathcal{F}_t)$ generated by the processes (υ_t), (ϑ_t), and (ν_t). The alive agent then decides her consumption level c_t, her savings in bonds b_t and stocks s_t, and her stock market participation status η_t (equal to 0, if she has never paid the participation cost before and therefore never participated).

Constraints. If the agent is dead at date t, she bequeaths all her wealth w_t:

$$w_t = R^f b_{t-1} + R_t^s s_{t-1}. \tag{12}$$

The stock holding s_{t-1} may be null if the agent has never participated in the stock market.

If the agent is alive, her resources at the beginning of the period consist of stock and bond payoffs plus the labor income y_t of the period. Resources cover consumption as well as the purchase of bonds and stocks. The agent can only invest in stocks if the participation cost, F, has been paid in period t or before. The budget constraint of the alive agent at date t can then be expressed as follows:

$$c_t + b_t + s_t + F 1_{\eta_t=1} 1_{\eta_{t-1}=0} = y_t + R^f b_{t-1} + R_t^s s_{t-1}, \tag{13}$$

where $1_{\eta_s=1}$ is an indicator function equal to 1 if $\eta_s = 1$ and 0 otherwise. Stocks can only be bought when the participation cost has been paid, neither asset can be sold short, and consumption must be strictly positive:

$$s_t > 0 \text{ iff } 1_{\eta_t=1} = 1, \tag{14}$$

$$b_t \geq 0 \text{ and } s_t \geq 0, \tag{15}$$

$$c_t > 0. \tag{16}$$

A feasible allocation is a sequence of choices $(c_t, b_t, s_t, \eta_t)_{t\geq 0}$ satisfying the constraints (12)–(16). The set of feasible allocations is denoted $\mathcal{A}$.

Regarding initial conditions, we assume without loss of generality that $\eta_{-1} = 0$ and $b_{-1} \geq 0$ and $s_{-1} \geq 0$ are exogenous values. Equations (12) and (13) can be assumed to hold at date $t = 0$.

3.2 Preferences

We denote by $u(c_t)$ the instantaneous felicity the agent gets when being alive and consuming c_t and by $v(w_t)$ the felicity she derives when being dead and bequeathing the amount w_t. Preferences are separable over time and future instantaneous felicities are discounted by a factor $\beta \in (0, 1)$ representing the agent's exogenous time preference.

Regarding risk preferences, we consider recursive utilities à la Kreps and Porteus (1978). Agents value the certainty equivalent of future utility streams. More precisely, for an increasing concave function $\Phi : \mathbb{R} \to \mathbb{R}$, the utility U_t at date t

expresses as follows:[4]

$$U_t = (1-\beta)u_t + \beta\Phi^{-1}\left(E_t^{\mathcal{F}\times\mathcal{G}}\left[\Phi(U_{t+1})\right]\right), \text{ with } u_t = \begin{cases} u(c_t), & \text{if the agent is alive at } t, \\ v(w_t), & \text{if the agent is dead at } t. \end{cases} \tag{17}$$

In the above equation, $E_t^{\mathcal{F}\times\mathcal{G}}[\cdot]$ is the conditional expectation operator with respect to the information available at date t. Formally, the information is the filtration $(\mathcal{F}_t \otimes \mathcal{G}_t)_{t\geq 0}$, where $(\mathcal{G}_t)_{t\geq 0}$ is the filtration generated by the independent mortality process. The factor $1-\beta$ multiplying u_t in equation (17) is a normalization that simplifies the expression of the utility of a dead agent (see equation (18) below).

In such models, if there were no uncertainty, the utility U_t would be independent of the function Φ and the recursion (17) would reduce to $U_t = (1-\beta)u_t + \beta U_{t+1}$. We thus have a possible separation between preferences over certain consumption streams—determined by the functions u, v, and the scalar β—and risk preferences, determined by the function Φ. A more concave Φ implies lower certainty equivalents $\Phi^{-1}\left(E_t^{\mathcal{F}\times\mathcal{G}}\left[\Phi(U_{t+1})\right]\right)$ and therefore greater risk aversion.

Our specification of recursive preferences nests the most common cases, including the additive specification, the Epstein and Zin (1989) isoelastic specification, and the risk-sensitive specification introduced by Hansen and Sargent (1995) in their work on robustness. In Section 4, we make precise the functions Φ which correspond to each of these different specifications. Our results make it possible to discuss the impact of these specifications on saving decisions.

3.3 Agent's program

We can now write the agent's program recursively by taking advantage of the structure of preferences. We denote by U_t^D the intertemporal utility at date t of a dead agent and by U_t^A that of an alive agent. The agent derives felicity of bequests only in the period she dies. After that, felicity is constant and equal to $v(0)$. From the recursive formulation (17), we deduce that there is no actual optimization and

[4] Formally speaking, preferences are defined over the set of temporal lotteries, allowing for preferences for late or early uncertainty resolution. See Epstein and Zin (1989) or Wakai (2007) for a formal treatment.

that the intertemporal utility of a dead agent can be expressed as:

$$U_t^D(w_t) = (1-\beta)v(w_t) + \beta v(0). \tag{18}$$

While alive, the agent maximizes her intertemporal utility by choosing a feasible allocation $(c_t, b_t, s_t, \eta_t)_{t\geq 0}$ in the set $\mathcal{A}$. The utility U_t^A of the alive agent at age t depends on four state variables: the beginning-of-period holdings in stocks s_{t-1} and bonds b_{t-1}, the persistent component of stochastic labor income, π_{t-1}, and the stock market participation status $\eta_{t-1} \in \{0,1\}$. The latter is discrete, while the three former ones are continuous. From the recursive formulation (17) and feasibility constraints (12)–(16) and using the fact that the mortality risk is assumed to be independent of other risks, the program of an alive agent at date t can be expressed as follows:

$$\begin{aligned} U_t^A(s_{t-1}, b_{t-1}, \eta_{t-1}, \pi_{t-1}) &= \max_{(c_t, s_t, b_t, \eta_t)\in\mathcal{A}} (1-\beta)u(c_t) \\ &+ \beta\Phi^{-1}\left(p_t E_t[\Phi(U_{t+1}^A(s_t, b_t, \eta_t, \pi_t))] + (1-p_t)E_t[\Phi(U_{t+1}^D(w_{t+1}))]\right), \end{aligned} \tag{19}$$

where $E_t[\cdot]$ is the expectation for an alive agent with respect to the filtration $\mathcal{F}$ (i.e., made of all past shock realizations but death). Note that we distinguish it from the expectation $E_t^{\mathcal{F}\times\mathcal{G}}[\cdot]$ with respect to the whole information $(\mathcal{F}_t \otimes \mathcal{G}_t)_{t\geq 0}$ (including death information). Recall that the program (19) has a finite horizon since there exists a maximal age for the agent, T_M.

3.4 Value of life

Applying the definition in equation (1), we derive the following expression for the VSL at age t:

$$VSL_t = \frac{\frac{\partial U_t^A}{\partial p_t}}{\frac{\partial U_t^A}{\partial c_t}} = \frac{\beta}{1-\beta} \frac{E_t\left[\Phi\left(U_{t+1}^A\right) - \Phi\left((1-\beta)v(w_{t+1}) + \beta v(0)\right)\right]}{u'(c_t)\Phi'\left(\frac{1}{\beta}\left(U_t^A - (1-\beta)u(c_t)\right)\right)}. \tag{20}$$

We will use this expression to calibrate the model to empirical estimates of the VSL, cf. Section 5.

4 Specifications of utility functions

In this section we specify the functional forms for felicity functions u and v and for the aggregator Φ. Our specification choices are constrained by the fact that our model must yield reasonable predictions as to the impact of mortality and the value of life. Moreover, we focus on standard functional forms whenever possible. Specifically, we assume a constant IES and cover risk-sensitive preferences of Hansen-Sargent and Epstein-Zin preferences, highlighting advantages and drawbacks of each.

4.1 Felicity function specification

We begin with specifying u and v. We assume that the agent has a constant IES, which means that $-\frac{u'(c)}{c\,u''(c)}$ is constant. This implies that u is equal, up to an affine transformation, to:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma} & \text{if } \sigma \neq 1, \\ \ln(c) & \text{if } \sigma = 1, \end{cases} \tag{21}$$

where the parameter $\sigma > 0$ is the inverse of the IES. The above specification is such that $u(1) = 0$. It embeds therefore a normalization assumption, which is without generality loss. The case $\sigma = 1$ is obtained by continuity from the general case.

The felicity derived from bequeathing wealth w is assumed to have the following functional form:

$$v(w) = \begin{cases} -\bar{v} + \frac{\theta}{1-\sigma}\left[(\overline{w} + w)^{1-\sigma} - \overline{w}^{1-\sigma}\right] & \text{if } \sigma \neq 1, \\ -\bar{v} + \theta \ln\left(\frac{\overline{w}+w}{\overline{w}}\right) & \text{if } \sigma = 1, \end{cases} \tag{22}$$

where σ is the inverse of the IES used in the expression (21) of the felicity of u, $\overline{v} = -v(0) \in \mathbb{R}$, $\theta \geq 0$, and $\overline{w} \geq 0$ are parameters that we discuss below. As for u, the case $\sigma = 1$ in (22) is obtained by continuity from the general case.

We can distinguish two components in the specification of v in equation (22). The first one is the term $(-\overline{v})$, which corresponds to the difference in felicity between being dead and bequeathing nothing and being alive and consuming one unit. This difference is negative if death is not as good as being alive with one unit of consumption, the constant $\overline{v}$ being then positive. A higher (resp. lower) value of $\overline{v}$ will be associated with a higher (resp. lower) valuation of being alive, compared to being dead. The value of $\overline{v}$ thus strongly connects to the value of life. The second part, $\frac{1}{1-\sigma}\left[(\overline{w}+w)^{1-\sigma}-\overline{w}^{1-\sigma}\right]$, measures the contribution of bequest to post-mortem felicity. This extra felicity derived from bequest is assumed to be continuous in zero, increasing in the amount of bequest, and exhibiting bounded and decreasing marginal felicity. The rationale for this functional expression is the following one. Heirs may already have individual resources at their disposition, summarized by the quantity $\overline{w}$, and they enjoy bequest in addition to these resources $\overline{w}$. The felicity derived by heirs from bequest is proxied by the quantity $\frac{1}{1-\sigma}\left[(\overline{w}+w)^{1-\sigma}-\overline{w}^{1-\sigma}\right]$.[5] The agent values the felicity of her heirs with the weight θ that can therefore be interpreted as the intensity of the altruistic bequest motive. With $\overline{w}>0$, bequests are a luxury good, as reported in the data (e.g., in Hurd and Smith, 2002). Indeed, the derivative $v'(0)$ is finite, so that agents bequeath only when their wealth is large enough. This functional form has been chosen for example in De Nardi (2004), De Nardi et al. (2010), Ameriks et al. (2011), and Lockwood (2012, 2014).

We now discuss the two functional forms we consider for the function Φ.

4.2 Risk-sensitive preferences

First, we consider risk-sensitive preferences:

$$\Phi(u)=\begin{cases}-\frac{1}{k}\left(\exp(-ku)-1\right) & \text{if } k\neq 0,\\ u & \text{if } k=0,\end{cases} \tag{23}$$

where k is a constant driving risk aversion. The case $k=0$ corresponds to the usual additive model and is obtained by continuity of the general case. For an agent endowed with risk-sensitive preferences to be more risk averse than in the usual additive model, we need to assume that $k>0$. Risk-sensitive preferences have been introduced in Hansen and Sargent (1995) and axiomatized in Strzalecki (2011). As

[5]The proxy is exact if (i) heirs have the same IES as the donator and (ii) heirs fully annuitize their wealth.

shown in Bommier, Kochov and LeGrand (2017), this is the only functional form Φ for which preferences represented by the utility function in recursion (17) are monotone. The monotonicity of preferences has to be understood as monotonicity with respect to first-order stochastic dominance. Such a monotonicity means that if two uncertain consumption streams are available and the first one is preferred to the other one in any possible state of the world, the former will always be preferred to the latter. This is distinct from and not implied by the fact that more certain consumption is preferred to less, which in our setup is equivalent to an increasing felicity function u. Moreover, as proved in Bommier and LeGrand (2014), risk-sensitive preferences are well-ordered with respect to risk aversion, both "in the large" (i.e., in terms of willingness-to-pay to eliminate all risks) but also "in the small" (i.e. in terms of willingness-to-pay for marginal risk reductions). This last aspect is important when addressing problems where complete risk elimination is not possible, or simply not optimal, as is the case in the portfolio choice that we study.

4.3 Epstein-Zin preferences

Second, we consider Epstein-Zin isoelastic preferences, which correspond to the following functional form for Φ:

$$\Phi(u) = \begin{cases} \frac{1}{1-\gamma}(1+(1-\sigma)u)^{\frac{1-\gamma}{1-\sigma}} - \frac{1}{1-\gamma}, & \text{if } \gamma \neq 1 \text{ and } \sigma \neq 1, \\ \frac{1}{1-\sigma}\ln(1+(1-\sigma)u), & \text{if } \gamma = 1 \text{ and } \sigma \neq 1, \\ \frac{1}{1-\gamma}e^{(1-\gamma)u} - \frac{1}{1-\gamma}, & \text{if } \gamma \neq 1 \text{ and } \sigma = 1, \\ u, & \text{if } \gamma = 1 \text{ and } \sigma = 1, \end{cases} \tag{24}$$

where $\gamma \in \mathbb{R}$ and $1 + (1-\sigma)u \geq 0$.[6] Whenever $\gamma = \sigma$, we get $\Phi(u) = u$ and Epstein-Zin preferences are additive. It is also well-known from Tallarini (2000), and directly visible from the last two lines of (24), that, when $\sigma = 1$, Epstein-

[6]Epstein-Zin preferences are often introduced with a different, but equivalent normalization for the function u (e.g., using $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ instead of $u(c) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma}$), and therefore different functions Φ (the constant 1 being no longer needed). Our (equivalent) approach has however the advantage to have the cases $\sigma = 1$ or $\gamma = 1$ directly obtained as limit cases of the others, while keeping $\overline{v}$ or $\overline{w}$ independent of σ and γ. This normalization choice has no impact on our results.

Zin preferences coincide with risk-sensitive preferences. Thus, the cases where $\sigma = 1$ are already addressed with risk-sensitive preferences and do not need further consideration. We will therefore exclude them whenever we refer to Epstein-Zin preferences below. For $\sigma \neq 1$, the constraint $1+(1-\sigma)u \geq 0$ is not trivial. It holds whenever the agent is alive, since we have $1 + (1 - \sigma)u(c) = c^{1-\sigma}$, but imposes constraints on the felicity of bequest defined in equation (22).[7]

Isoelastic Epstein-Zin preferences are very popular in macroeconomics and finance, one of their main advantages being that they usually provide a homothetic specification, which affords convenient tractability. Note that this is not the case in our setup, where Epstein-Zin preferences are not homothetic. The reason is not the normalization that we made in equation (24), but stems from our choice of leaving u_d unconstrained, so as to be able to match plausible values of life.

An inconvenient aspect of Epstein-Zin preferences is that they are not monotone.[8] This may yield unintuitive conclusions in some cases, even though it turns out that this is not the case in the current paper, which relies on a non-homothetic specification.[9]

5 Calibration and computation

In this section, we first give an overview of our calibration strategy and the computational experiment. We then discuss the resulting parametrization of our baseline economy in detail. The last section discusses the main aspects of solving the model computationally. Robustness checks on the calibration are discussed in Section 6.2.

5.1 Calibration strategy and computational experiment

Our calibration shares many common aspects with the related literature but differs mainly in that we target the VSL explicitly. Given a realistic value of life, the

[7]The constraint stems from the fact that for Epstein-Zin preferences, felicity of being alive and of being dead must always be of the same sign. See Bommier, Harenberg and LeGrand (2017), Section 4.3, for a more in-depth discussion.

[8]Cf. Bommier, Kochov, LeGrand (2017).

[9]See also Section 6.3 for further discussion on serious pitfalls that arise when using homothetic Epstein-Zin specifications.

objective of the computational experiment is to highlight the impact of risk aversion. To this aim, we consider three agents: one with standard additively separable preferences, one with Epstein-Zin preferences corresponding to the aggregator (24), and one with risk-sensitive preferences corresponding to the aggregator (23).[10] We will henceforth refer to the three agents as the additive, the Epstein-Zin (denoted by superscript EZ), and the risk-sensitive agent (denoted by superscript RS). Importantly, we calibrate only the additive agent to the data. The other two agents take the same parameter values as the additive agent and differ only in that they have a higher degree of risk aversion ($k > 0$ for risk-sensitive preferences and $\gamma > \sigma$ for Epstein-Zin preferences).

We now describe our strategy for calibrating the additive agent; the resulting parameter values are discussed in the following sections. We set the IES, $\frac{1}{\sigma}$, to a standard value. We then jointly calibrate the discount factor, β, the bequest motive, θ, the endowment of the offspring, $\bar{w}$, the life-death utility gap, $\overline{v}$, and the stock market participation cost, F, to match the following five targets. The first target is an estimate of the value of life at age 45, VSL_{45}, as defined in equation (20). Targeting this is central to our exercise. The other targets are average assets at age 45, median bequests at age 90, the top quintile of bequests at age 90, and average stock market participation rate at age 60. Note that, since we have continuous shocks to labor and asset income in the model, there is a distribution of outcomes at each age, which is equivalent to a distribution of ex-ante identical agents. Thus, the empirical targets have an exact counterpart in the model.[11] The remaining parameters are set to values that are taken directly from available data or related studies. Table 1 summarizes the parameter values. The last column indicates whether the parameter is exogenously set ("stage 0") or endogenously calibrated ("stage 1").

After having calibrated the additive agent, we set the risk aversion of the Epstein-Zin agent to a slightly higher value $\gamma > \sigma$ (indicated by "stage 2" in Table 1), while keeping all other parameters the same. This isolates how risk

[10] Recall from Section 4 that the additively separable case is nested in Epstein-Zin preferences when $\gamma = \sigma$, and in risk-sensitive preferences when $k = 0$.

[11] This is particularly relevant for the VSL, which by definition applies only to a large population, cf. our discussion is Section 2.1.

aversion impacts lifecycle savings and portfolio choices. Similarly, in "stage 3" we increase risk aversion k of the risk-sensitive agent by setting $k > 0$. More precisely, we calibrate k to produce the same average savings at age 64 as the Epstein-Zin agent, i.e., such that $E_0[s_{64}^{RS} + b_{64}^{RS}] = E_0[s_{64}^{EZ} + b_{64}^{EZ}]$. We do this, because we want the increases in risk aversion to be of similar magnitude, whether it is achieved with risk-sensitive or Epstein-Zin preferences. Both the Epstein-Zin and risk-sensitive agents are more risk averse than the additive agent but are not comparable with each other in terms of risk aversion.

Our computational implementation matches all targets nearly exactly, with deviations of less than one percent. See Appendix A for details.

5.2 Demographics

A model period corresponds to one year. Agents start being economically active in the model at working age, which we set to 20. They exogenously retire at the fixed age of 65, which corresponds to the statutory retirement age in the U.S. Mortality rates are taken from the Human Mortality Database for the USA for 2007 and are displayed in Appendix B. The maximum biological age is capped at 100.

5.3 Preferences

The IES is set to 0.5, a common value in the literature, so that its inverse is $\sigma = 2$. As previously mentioned, the parameters $\overline{v}$, β, $\bar{w}$, and θ are jointly calibrated so that the additive agent has a VSL at age 45, assets at age 45, and median and top quintile of bequests at age 90 that match their empirical counterparts. For VSL, we target US$ 6.5 million, which is in the middle of available estimates, cf. Viscusi and Aldy (2003). For average individual assets, we target US$ 100 000, which is consistent with Census data. For bequests we match a median of US$ 30 000 at age 90 and a top quintile of US$ 130 000, which are in line with the values reported by De Nardi, French and Jones (2010). This yields $\overline{v} = 24.33$, $\beta = 0.959$, $\bar{w} = 0.247$, and $\theta = 11.01$.

For the Epstein-Zin and risk sensitive agents, we use the same set of parameters as in the additive model, at the exception of the risk aversion parameters. The

Table 1: Parameterization in baseline economy

Parameter	Value	Source or empirical target	Stage
Demographics			
Biological age at $t = 0$	20	Age at labor force entry	0
Model age at retirement, T_R	45	SSA statutory retirement age of 65	0
Model age maximum, T_M	80	Biological age of 100	0
Cond. survival rates, $\{p_t\}$	cf. App. B	Human Mortality Database, U.S. 2007	0
Preferences			
Inverse IES, σ	2.0		0
Risk aversion, Epstein-Zin, γ	3.0		2
Risk aversion, risk-sensitive, k	0.079	Assets of EZ agent at age 64	3
Life-death utility gap, $\overline{v}$	24.33	VSL at age 45 of US$ 6.5 million	1
Discount factor, β	0.959	Assets at age 45 of US$ 100 000	1
Strength of bequest motive, θ	11.01	Median bequests at age 90 of US$ 35 000	1
Bequest luxury good strength, $\overline{w}$	0.247	Top quintile of bequests at age 90 of US$ 150 000	1
Endowments			
Average wage, $\bar{y}$	US$ 32 191	Net compensation 2007, SSA	0
Pension, y^R	40%$\times\bar{y}$	Average replacement rate	0
Age productivity, $\{\mu_t\}$	cf. App. B	Earnings profiles 2007, PSID	0
Labor income autocorrelation, ρ	0.988	Guvenen (2009)	0
Var. of persistent shocks, σ_v^2	0.015	Guvenen (2009)	0
Correlation with stock return, κ_v	0.150	Gomes and Michaelides (2005)	0
Var. of transitory shocks, σ_ϑ^2	0.061	Guvenen (2009)	0
Asset Markets			
Gross risk-free return, R^f	1.02	Bond return (Shiller)	0
Equity premium, ω	0.02		0
Stock volatility, σ_ν	0.18	Shiller data	0
Participation cost, F	83.2% of $\bar{y}$	Participation rate at age 60 of 50%	1

Notes: In the last column: stage 0 parameters set exogenously, stage 1 parameters jointly calibrated to the targets for agent with additive preferences, stage 2: risk aversion parameter for EZ agent set exogenously, stage 3: risk aversion parameter for RS agent calibrated to respective target.

Epstein-Zin specification assumes $\gamma = 3$, affording a moderate increase in risk aversion as compared to the additive case (obtained for $\gamma = 2$). For the risk-sensitive case, we calibrate the risk aversion parameter to match the same average savings as the Epstein-Zin agent at the age of 64, which yields $k = 0.079$.

5.4 Endowments

The average wage, $\bar{y}$, is set to US$ 32 191, which is the net compensation reported by the US social security administration. Pensions, y^R, are set to 40 percent of the average wage, in line with the average U.S. social security replacement rate, cf. Rettenmaier and Saving (2006). The deterministic age-productivity profile is taken from Harenberg and Ludwig (2015), who compute it from PSID data using the method of Huggett, Ventura and Yaron (2011). The values are displayed in the calibration Appendix B.

The values for the stochastic income process are taken from Guvenen (2009), who reports an autocorrelation $\rho = 0.988$, a variance of persistent shocks of $\sigma_{\upsilon}^2 = 0.015$, and a variance of the temporary income shocks of $\sigma_{\vartheta}^2 = 0.061$. Following Gomes and Michaelides (2005) we set the correlation of the persistent shocks with stock returns to $\kappa_{\upsilon} = 0.15$. We assume that the agent enters the model with no assets, $s_{-1} = 0$ and $b_{-1} = 0$.

5.5 Asset markets

The gross risk-free return is set to the average bond return over the last 50 years in the data of Robert Shiller, $R^f = 1.02$.[12] We set the equity premium to a low value of $\omega = 2\%$, as agents in our calibration have a low risk aversion and we do not aim to match the equity premium. Stock volatility is $\sigma_{\nu} = 0.18$, again measured in Robert Shiller's data over the last 50 years. Participation cost is calibrated (jointly with the preference parameters) to deliver a stock market participation rate for the additive agent of 50% at age 60, which is in the range reported by Alan (2006). The required cost of 82.2% of average, annual income is very high because it is

[12] Robert Shiller's data are freely available at `http://www.econ.yale.edu/~shiller/data.htm`.

paid only once in life and agents in our baseline are not very risk-averse.[13]

5.6 Computational solution

We solve the model using value function iteration. The reason for that is that value function iteration can handle discrete choices such as our stock market participation decision.[14]

We treat the persistent component of the stochastic income process, π_t, as a continuous state variable, cf. equations (10) and (11). This allows us to approximate income risk more precisely than the usual discretization schemes à la Tauchen and Hussey (1991) and to take the cross-correlation with stock returns into account. Expectations are evaluated using Gaussian quadrature.

The model is solved in terms of cash-at-hand (Deaton 1991), thereby reducing the state space by one dimension. That leaves us with a state space consisting of two continuous (cash-at-hand and persistent component of income) and two discrete variables (age and participation choice). To speed up the solution we interpolate the expected continuation value of the agent with two-dimensional cubic B-splines.

To compute average savings and consumption paths, we run stochastic simulations. To take into account the continuous nature of shocks in the simulations, we compute a histogram over cash-at-hand and iterate on that. This is more accurate than simulating a large number of agents, cf. Algan, Allais, Den Haan and Rendahl (2014). Average lifecycle profiles—which are our main object of interest—are then directly obtained from the distribution of cash-at-hand and the optimal policy function at each age.

The computational solution is parallelized and run on 24 cores. Further details on the solution method are provided in Appendix A.

[13]In Section 6.2, we study participation costs paid every time the agent invests in stock, instead of once per life. In that calibration, the costs fall to 3.7% of average income, which is closer to empirical estimates, cf. Fagereng, Gottlieb and Guiso (2017).

[14]Because of the stock market participation cost, the discrete participation choice introduces multiple discontinuities in the optimal continuous choices, cf. Campanale, Fugazza and Gomes (2015). That is why we opt for a robust algorithm.

6 Results

We first describe the outcomes of the model, as calibrated in Section 5. Then we report the results of several robustness checks. Finally, we explain the relationship of our findings to the previous literature.

6.1 Lifecycle profiles

To present our results, we focus on average lifecycle profiles for agent choices. Each profile corresponds to the profile conditional on the agent surviving until the maximal age, averaged over all possible realizations for the income and investment risks. Thus, at each age, there is a whole distribution of agent's wealth, which correspondingly implies a distribution of consumption, VSL, etc.[15]

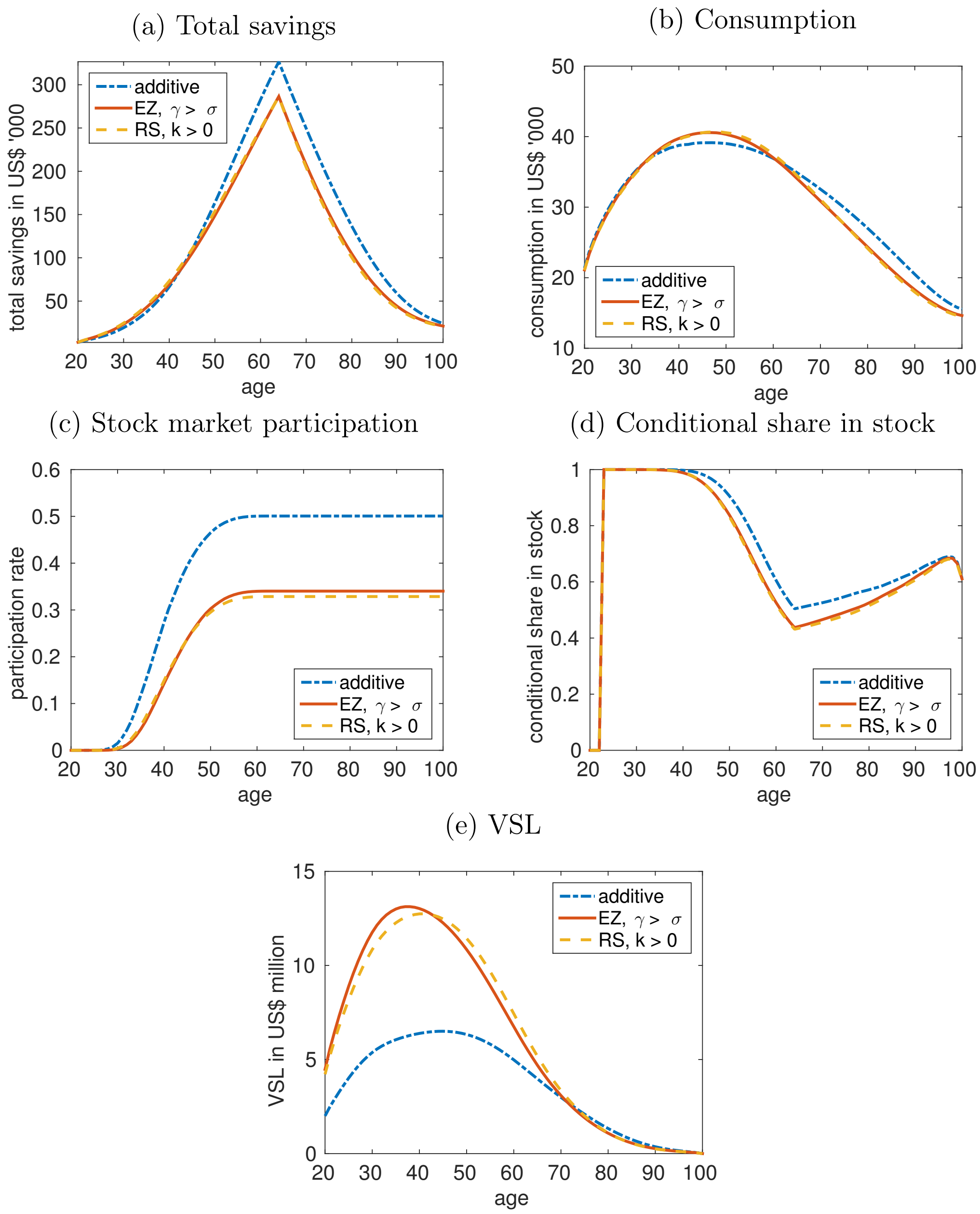
The panels in Figure 1 display the lifecycle profiles for the additive, risk-sensitive, and Epstein-Zin agents. Let us first focus on total lifecycle savings, which are shown in panel (a). Overall, the shape of the saving profile is very similar for the three agents. As is typical of such lifecycle models, agents build up savings during their working age, until they reach the exogenous retirement age of 65, and then gradually decumulate their savings.[16] However, the agents differ markedly in the level of savings they accumulate. The additive agent saves much more on average than both the Epstein-Zin and risk-sensitive agents.[17] The reason is the one explained in Section 2.3, namely that risk aversion amplifies the role of mortality risk in the discount rate, if the value of life is positive. In our computational experiment, the VSL is calibrated to be positive, cf. Figure 1e, and both the Epstein-Zin and the risk-sensitive agents are more risk averse than the additive agent by construction. Therefore, their discount rates are higher than that of the additive agent, and they save less. Crucially, this effect is strong enough to overturn the higher savings due to higher prudence of more risk averse agents. That is, the "carpe diem" consumption motive dominates the precautionary savings motive.

[15] Cf. Section 5.6 on details of how the distribution and lifecycle profiles are computed.

[16] For the additive agent, average savings at age 90 are significantly larger than the targeted median bequest of US$ 30 000, because the bequest distribution is skewed. Recall that we partially match the skewness by targeting also the top quintile of bequests at age 90.

[17] While in our baseline calibration the Epstein-Zin and the risk-sensitive agents have nearly identical lifecycle savings and consumption profiles, this need not generally be the case.

Figure 1: Lifecycle profiles for benchmark economy



The corresponding lifecycle consumption profiles are shown in panel (b). They are, of course, consistent with the lifecycle saving profiles and they are hump-shaped. The risk averse agents consume more than the additive agent at earlier ages (between ages 30 and 60) and less at older ages (greater than 60). A greater risk aversion tends therefore to increase consumption at earlier age and to decrease it a later age. Again, the reason is that risk aversion amplifies the role of mortality in the discount rate. A more risk averse agent is more impatient and therefore consumes early. This is the "carpe diem" consumption motive, which only obtains if life is valued positively.

Let us now turn to the lifecycle stock market participation rates, displayed in panel (c). The participation rates are an increasing function of age, because, in our baseline, the cost to access the stock market is paid only once in life. Importantly, the more risk averse agents, i.e., the Epstein-Zin and risk-sensitive agents, on average participate less in the stock market. While it is intuitive that more risk averse agents choose a smaller exposure to the investment risk, this result stands in contrast to the previous literature. The reason for the different finding is that we assume a positive VSL, implying that the more risk averse agents accumulate less assets and are therefore less willing to pay the participation cost.[18]

Conditional on participation, more risk averse agents hold a smaller share of their savings in stocks at all ages, cf. panel (d). So more risk averse agents consistently choose to take less risk: first, by participating less in the stock market, and second, by holding a smaller conditional risky share.

Last, but not least, panel (e) of Figure 1 plots the lifecycle profile of the VSL. As explained above, it is calibrated such that the additive agent has an average VSL of approximately US$ 6.5 million at age 45. It is positive at all ages, which is in contrast to the previous household finance literature (see Section 6.3 for further discussion). The hump-shape follows from the interplay of two forces, consumption and remaining life expectancy. First, the consumption increase dominates the decrease in remaining life expectancy so that VSL increases and peaks close to prime age. After age 47, the VSL decreases because of the decreases in consumption and in expected remaining life-duration. As with the other profiles, there is a distri-

[18] We discuss the relationship to the previous literature in more detail in Section 6.3.

bution over the VSL at each age, with larger wealth realizations corresponding to a larger VSL and lower ones to a smaller VSL. The VSL of the Epstein-Zin and the risk-sensitive agent are substantially larger until age 72 because of their higher risk aversion, cf. Equation (20). After age 72, the VSL of the additive agent is slightly higher, because of her higher consumption stream at older ages.

6.2 Robustness of results

In this section we summarize the results of a variety of robustness checks, with details relegated to Appendix C.

First, we study the sensitivity with respect to the VSL by targeting values at the upper and the lower end of available estimates. When we target a high VSL of US$ 15 million, our baseline results are clearly confirmed: more risk averse agents save less, participate less in the stock market, and—conditionally on participating—invest a smaller share in stock. However, when we target a low VSL of US$ 1 million, results for lifecycle savings are less clear. The reason is that the "carpe diem" motive is much weaker and barely dominates the precautionary saving motive. Nonetheless, risk aversion still clearly lowers stock market participation and the conditional risky share.

Next, we study the importance of the bequest motive by making it ineffective, so that agents do not derive any value from bequests. The main difference in the results is that the lifecycle savings profile falls to zero at the maximum age, as agents deplete their assets completely towards the end of life. This results in a conditional risky share that increases more strongly after retirement.[19] But our main results are still clearly confirmed: more risk averse agents save less, participate less in the stock market, and—at least until age 82—hold a (weakly) smaller conditional risky share.

We also considered other forms of stock market participation cost, beyond the once-in-a-life considered in our baseline: a cost proportional to the stochastic income component as in Gomes and Michaelides (2005)—to be interpreted as an

[19] The fact that lower wealth realizations imply a larger share of stock holdings is a common feature of lifecycle models of portfolio choice. As a consequence, matching the conditional risky share at older ages to the data is difficult in this kind of models. For example, Fagereng et al. (2017) need disaster shocks to reduce the conditional risky share at old age.

opportunity cost—and a cost paid every time the agent invests in stock, as in Fagereng et al. (2017). The most relevant difference in the results is that with a cost paid every time, the lifecycle profile of participation becomes hump-shaped, which is somewhat closer to the data (cf. Alan 2006 or Fagereng et al. 2017). In addition, the participation cost drops to around US$ 1000, a value closer to those papers. Apart from that, our main results are again clearly confirmed for both types of participation cost.

Finally, we used other estimates for the labor income process that can be found in the literature. Since that had only negligible effects, we do not report the results.

6.3 Relation to previous literature

Our calibrated lifecycle model provides new insights on lifecycle behavior. In particular, we find that risk aversion decreases savings and stock market participation. This contrasts with the predictions of Gomes and Michaelides (2005, 2008) and many related papers in the household finance literature. The divergence in predictions is directly related to value of life matters. In the current paper, our model was designed to fit empirical estimates of the VSL. Therefore, by construction the model provides a large and positive VSL. In that respect, our approach differs from the standard one in household finance, which consists in focusing on a homothetic version of Epstein-Zin preferences, without paying attention to the implications regarding the value of life.

In the homothetic version of the Epstein-Zin specification, the utility conditional on being alive at time t, denoted V_t, can be expressed as

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\left(E_t\left[p_t V_{t+1}^{1-\gamma} + (1-p_t) b w_{t+1}^{1-\gamma}\right]\right)^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{1-\sigma}}, \tag{25}$$

where, as before, c_t is consumption in period t, p_t is the probability of remaining alive in period $t+1$, w_{t+1} is the amount bequeathed in case of death, σ is the inverse of the IES, and γ is the coefficient of relative risk aversion.[20] The parameter b determines the intensity of the bequest motive. This model was not designed for fitting a specific VSL but rather chosen for its tractability. Equation (25)

[20] A formal derivation of equation (25) can be found in the appendix of Gomes, Michaelides, and Polkovnichenko (2009).

nevertheless implicitly assumes a specific value of life given by:

$$VSL_t = \frac{\frac{\partial V_t}{\partial p_t}}{\frac{\partial V_t}{\partial c_t}} = \frac{\beta c_t^{\sigma}}{1-\beta} \frac{E_t[V_{t+1}^{1-\gamma}] - bE_t[w_{t+1}^{1-\gamma}]}{1-\gamma} \left(E_t\left[p_t V_{t+1}^{1-\gamma} + (1-p_t) b w_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma-\sigma}{1-\gamma}},$$

which can be positive or negative. If $\gamma > 1$, as is assumed in the papers mentioned above, a positive value of life is obtained only if $b > \frac{E_t[V_{t+1}^{1-\gamma}]}{E_t[w_{t+1}^{1-\gamma}]}$. The results of Gomes and Michaelides (2005, 2008) indicate that this condition does not hold (at least not always) in their simulations. In particular, a negative value of life is systematically obtained when there is no bequest motive ($b = 0$), a case that Gomes and Michaelides consider in several instances, with no significant impact on their qualitative findings about the relationship between risk aversion and savings. As explained in Section 2.3, with a negative value of life, the rate of time discounting is underestimated, the bias being amplified by risk aversion. That is, for the case of a negative VSL, risk aversion is found to lower the discount rate, and hence to increase savings, providing a conclusion which is opposite to ours.[21] The difference in saving behavior eventually generates differences in the propensity to pay the stock market participation cost. This explains why they find that more risk averse agents tend to participate more frequently in the stock market, while we obtain the converse.

The fact that the homothetic Epstein-Zin specification with $\gamma > 1$ and no bequest motive provides a negative value of life was discussed in the value of life literature, in particular in Hugonnier, Pelgrin, and St-Amour (2013) and Córdoba and Ripoll (2017). These papers suggested to solve the issue by constraining γ to be lower than one, and to interpret this parameter as a coefficient of mortality risk aversion that could be different from the coefficient of financial risk aversion, the latter being unconstrained. The suggested solution seems appealing, as it preserves homotheticity (and hence high tractability), but another fundamental problem arises then: it becomes impossible to accommodate the case where $\sigma > 1$ (i.e., IES < 1), which is considered as the most empirically relevant (see Havránek

[21]One should moreover notice that if specification (25) were to be used with $\gamma > 1$ and a parameter b large enough to generate positive values of life, we would obtain a framework where the intensity of bequest motive would increase the willingness-to-pay for mortality risk reduction. However, this would go against intuition, since deriving utility from bequest reduces the welfare gap between life and death.

2015 for a recent reference). Indeed, as is formally shown in Bommier, Harenberg and LeGrand (2017), when $\gamma < 1 < \sigma$ and mortality rates take plausible values, the recursive equation (25) has only one degenerate solution where utility is equal to zero in all circumstances.[22] Such a degenerate model is of course unusable for discussing optimal consumption-saving behavior, providing moreover counterintuitive conclusions when first-order conditions are used while ignoring convergence and degeneracy issues.[23] Working with the homothetic specification where $\gamma < 1$ requires therefore to also assume $\sigma < 1$. Although this is technically feasible, we preferred to give up homotheticity rather than to constrain the IES, $\frac{1}{\sigma}$, to be above one. Note moreover that models that assume $\gamma < 1$ and $\sigma < 1$, as that of Ponzetto (2003), predict that risk aversion positively contributes to the discount rate (thereby decreasing savings), in line with our results.

7 Conclusion

The notion of value of life is generally not discussed in the household finance literature. The current paper shows that—the value of life plays a key role in the relation between mortality, risk aversion, and time discounting. It is, therefore, a crucial determinant of lifecycle behavior, even when mortality is exogenous.

Once the value of life is set to a reasonable positive level, our most important finding is that risk aversion has a negative impact on savings. The agent wants to save less and consume more because of a "carpe diem" consumption motive that becomes stronger with risk aversion. This "carpe diem" motive quantitatively dominates the well-known precautionary saving motive, because mortality risk is, typically, the largest risk in life. But for this to obtain quantitatively, it is crucial to calibrate the welfare impact mortality risk correctly, which involves matching reasonable empirical estimates of the value of life. A complementary interpretation is that saving involves keeping resources for periods that are only lived in favorable odds (i.e., in case of a long life). Saving is thus like a bet that

[22]Bommier, Harenberg and LeGrand (2017) demonstrate the result under the assumption that $b = 0$ (no bequest motive), to stick to the assumptions of Hugonnier, Pelgrin, and St-Amour (2013) and Córdoba and Ripoll (2017). The proof, however, extends to the case $b > 0$.

[23]See Sections 2.4 and 2.5 in Bommier, Harenberg and LeGrand (2017).

pays in good outcomes (i.e., survival) and therefore a risk taking behavior. If we were to assume that having a long life is a bad outcome—that is, assuming a negative value of life—the logic would be reversed. Savings would then imply keeping resources for adverse realizations (long lives) and would become a risk reducing behavior. The result would then be that risk aversion increases savings, exactly because of the assumption of a negative value of life.

Regarding stock market participation, we find that risk aversion decreases participation. This comprises a direct risk aversion effect and an indirect wealth effect through the "carpe diem" motive. Finally, we also find that higher risk aversion reduces the conditional risky share, so that our results consistently conform to the intuition that risk aversion reduces the propensity to engage in risky behaviors.

These results have, of course, wide-ranging implications for questions surrounding lifecycle models, such as the value of social security or the annuity puzzle, among others, which we leave for future research. In particular, one of the implications of our study, is that relatively low level of savings could be explained by risk aversion. While the economic literature abounds of works arguing that observed saving behaviors have to reflect strongly myopic preferences or some form of irrationality, our analysis suggests on the contrary that saving little could just be a rational decision for risk averse agents who are well aware that life duration is uncertain. Of course, low saving levels typically result in having a majority of (surviving) elderly declaring that they failed to save enough. But this is not evidence of under-savings. If they could be questioned, those who died before retirement might well answer that they actually saved too much.

Appendix

A Details on the computational implementation

The model is solved by backward iteration on the value function, starting with the maximum age T_M. We choose this robust algorithm because it can handle the discontinuities introduced by discrete choices. The agent's problem is rewritten in terms of cash-at-hand, $\tilde{x}_t = y_t + R^f b_{t-1} + R^s_t s_{t-1}$, thereby reducing the state space by one dimension (Deaton 1991). Next, we recast the two control variables stock, s_t, and bond, b_t, as total savings, a_t, and the portfolio share invested in stock, α_t. This is useful because $\alpha_t \in [0, 1]$ due to the no-short-selling constraint on both assets.

The persistent component of the stochastic income process, π_t, is implemented as a continuous state variable, cf. equations (10) and (11). This allows us to approximate income risk more precisely than the usual discretization schemes à la Tauchen and Hussey (1991) and to take the cross-correlation with stock returns into account.[24] The state space therefore consists of two continuous variables ($\tilde{x}_t$, π_t) and two discrete variables (age t, participation indicator η_t). We discretize the continuous variables with $n_{\tilde{x}} = 2400$ and $n_\pi = 24$ points. Model age t runs from 20 to 100, and the participation indicator takes values 0 (no participation) and 1 (participation). The continuous controls are discretized to $n_a = 4000$ points for assets and $n_\alpha = 800$ points for portfolio share. This high resolution allows for high accuracy even in the presence of discontinuities in the optimal choices.

Expectations are evaluated using multi-dimensional Gauss-Hermite quadrature with 5 points for each of the three shocks (transitory and persistent income shocks and stock return shock). The correlation between the persistent income shock and the stock return shock is implemented as described in Miranda and Fackler (2002). We evaluate expected continuation utility off the grid points by interpolating with two-dimensional cubic B-splines.[25]

[24] See, e.g., Flodén (2008) and Galindev and Lkhagvasuren (2010). Also more recent methods like Kopecky and Suen (2010) cannot directly handle cross-correlated processes.

[25] It can be shown that the expected value function is twice differentiable in a backward recursion scheme with a finite horizon by invoking the generalized envelope theorem of Clausen and

To compute average savings and consumption paths, we run stochastic simulations. To take into account the continuous nature of shocks in the simulations, we compute a histogram over cash-at-hand and iterate the histogram forward over age. The continuous shocks are approximated again by Gauss-Hermite quadrature. This is more accurate than simulating a large number of agents, cf. Algan et al. (2014) or Young (2010). The computational solution is parallelized and we run it on 24 cores.

The calibration procedure is cast as a system of nonlinear equations. Let $\mathcal{T}$ denote the target statistics in the data and $\mathcal{P}$ the model parameters to be calibrated. For given $\mathcal{P}$, $\hat{\mathcal{T}}(\mathcal{P})$ are the model-generated statistics, which we get from the simulations. Then the calibration procedure tries to find a root of $\mathcal{T} - \hat{\mathcal{T}}(\mathcal{P}) = 0$. To solve this system of equations we use Broyden's multidimensional secant method with a relative stopping criterion of $\epsilon_{\mathcal{T}} = \frac{\mathcal{T}-\hat{\mathcal{T}}(\mathcal{P})}{\mathcal{T}+1} = 1.0 \times 10^{-2}$.

B Lifecycle productivity and mortality profiles

Productivity is obtained from Huggett, Ventura, and Yaron (2011), by identifying cohort effects and applying smoothing. They use PSID data from 1969-2004, reported in 1968 US\$.[26] The 5-year bins are interpolated with a third-order polynomial. Productivity before age 20 is set to zero, and after age 66 is linearly extrapolated (but not used in our model). Figure 2a displays the corresponding log productivity profile.

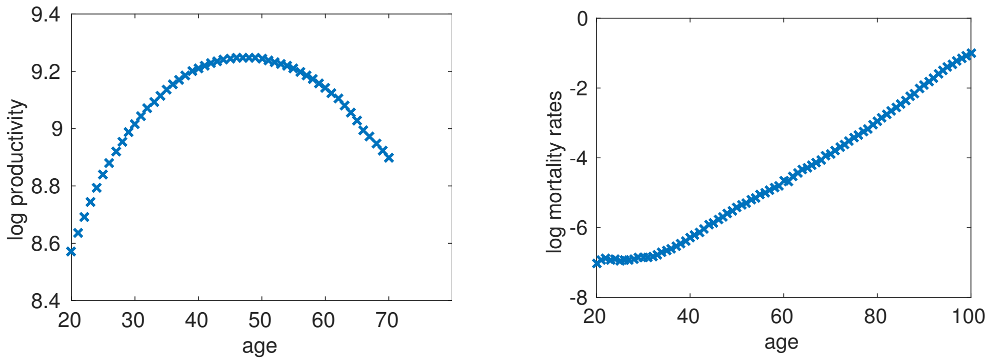
Mortality rates are taken from the Human Mortality Database for the U.S. in 2007 and displayed in Figure 2b. These are conditional mortality rates, i.e., the mortality rate for age 21 is the probability of not surviving to age 22 conditional on having reached age 21, for example. We use (male and female) average mortality rates, and set the mortality rate after the assumed maximum age to $p_{T_M} = 1$.

Strub (2012).

[26] Note that we multiply labor income with $\bar{y}$, thereby making sure that average labor income in 2007 is matched, cf. equation 10.

Figure 2: Lifecycle productivty and mortality profiles

(a) Deterministic lifecycle productivity (b) Mortality rates



C Additional results

We present the details of the other calibrations mentioned in Subsection 6.2. We show only the calibrated parameters, as all other parameters keep the values of the baseline displayed in Table 1. Unless explicitly stated, the targets and calibration strategy are the same as in the baseline, cf. Section 5.

C.1 High and low targets for the value of life

We here study the sensitivity of our results with respect to the value of life. To this aim, we first target a value at the upper end of available estimates and then a value at the very low end of available estimates.

C.1.1 Targeting a high VSL of US$ 15 million

We here report results when the target value for the VSL is US$ 15 million, which is at the upper end of available estimates and more than double the US$ 6.5 million of our baseline. The new values for the calibrated parameters are displayed in Table 2. Overall, they are very similar to the baseline values in Table 1, only the parameter that mostly influences the VSL, $\bar{v}$, approximately doubles to 50.48.

The resulting lifecycle profiles are displayed in Figure 3. The profiles are nearly indistinguishable from the baseline, with the minor difference that the conditional risky share increases a little bit more at the end of life, see panel (d). Our main findings are clearly confirmed: more risk averse agents save less, participate less

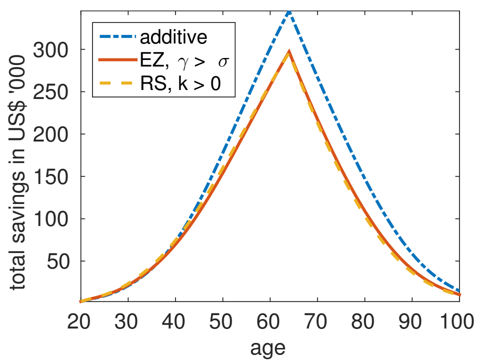
Table 2: Calibrated parameters with high VSL

Parameter	Value
Risk aversion, risk-sensitive, k	0.038
Life-death utility gap, $\overline{v}$	50.48
Discount factor, β	0.963
Strength of bequest motive, θ	15.72
Bequest luxury good strength, $\overline{w}$	0.466
Participation cost, F	90.6% of $\bar{y}$

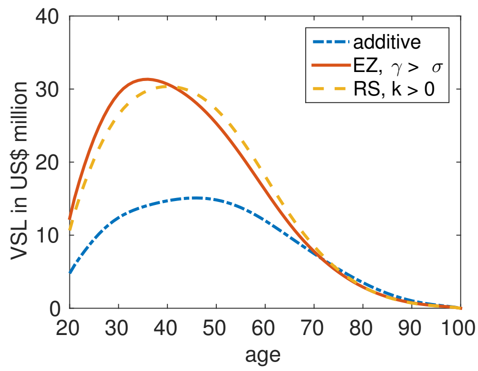
in the stock market, and—conditional on participating—invest less in stock.

Figure 3: Lifecycle profiles for economy with high VSL

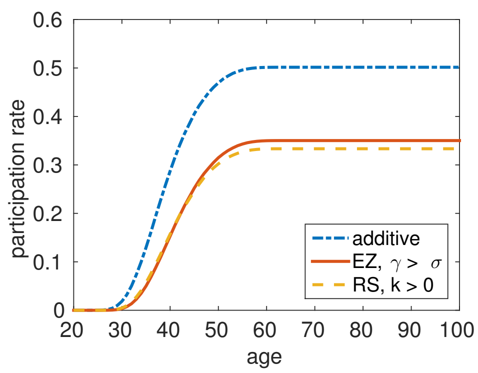
(a) Total savings



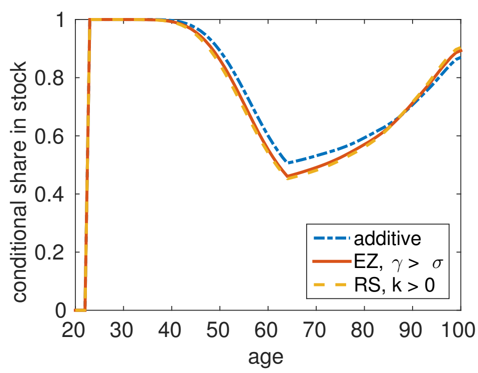
(b) VSL



(c) Stock market participation



(d) Conditional share in stock



C.1.2 Targeting a low VSL of US$ 1 million

We here report results when the target value for the VSL is only US$ 1 million, which is at the very low end of available estimates. The new values for the calibrated parameters are displayed in Table 3. There are some differences to the baseline values in Table 1, the reason being that now the "carpe diem" motive is much weaker, and the net effect of risk aversion is less clear. The parameter governing the VSL is, as expected, much smaller at $\bar{v} = 6.155$, while k is larger.

Table 3: Calibrated parameters with low VSL

Parameter	Value
Risk aversion, risk-sensitive, k	0.268
Life-death utility gap, $\overline{v}$	6.155
Discount factor, β	0.963
Strength of bequest motive, θ	56.06
Bequest luxury good strength, $\overline{w}$	2.976
Participation cost, F	88.8% of $\bar{y}$

The resulting lifecycle profiles are displayed in Figure 4. The saving profiles in panel (a) are very different from those in the baseline calibration, because now the "carpe diem" motive does not clearly dominate the precautionary savings motive anymore. A VSL of only US$ 1 million seems to be close to the point where both the "carpe diem" and the precautionary savings motives offset each other. As we can see in panel (b), more risk averse agents do not see such a large increase in VSL over the additive agent, in line with the fact that mortality is not experienced as such a big risk. Participation rates in panel (c) are very similar to the baseline. Finally, the conditional risky share in panel (d) shows a stronger increase after retirement for all agents, driven by the faster decumulation of assets as compared to the baseline.

C.2 No bequest utility

To understand the importance of the bequest motive, we here report results when the bequest motive is inactive. We do that by setting the parameter governing the strength of the bequest motive to zero, $\theta = 0$, and keeping the parameter governing the luxury nature of bequests, $\bar{w}$, at the value of the baseline.[27] The remaining calibration parameters, β, $\bar{v}$, F, and k are calibrated to the same targets as in the baseline calibration. Their new values are displayed in Table 4. Compared to the baseline values in Table 1, β, $\bar{v}$, F, and k change very little.

[27]When $\theta = 0$, the parameter $\bar{w}$ is inactive as long as it takes a finite, positive value.

Figure 4: Lifecycle profiles for economy with low VSL

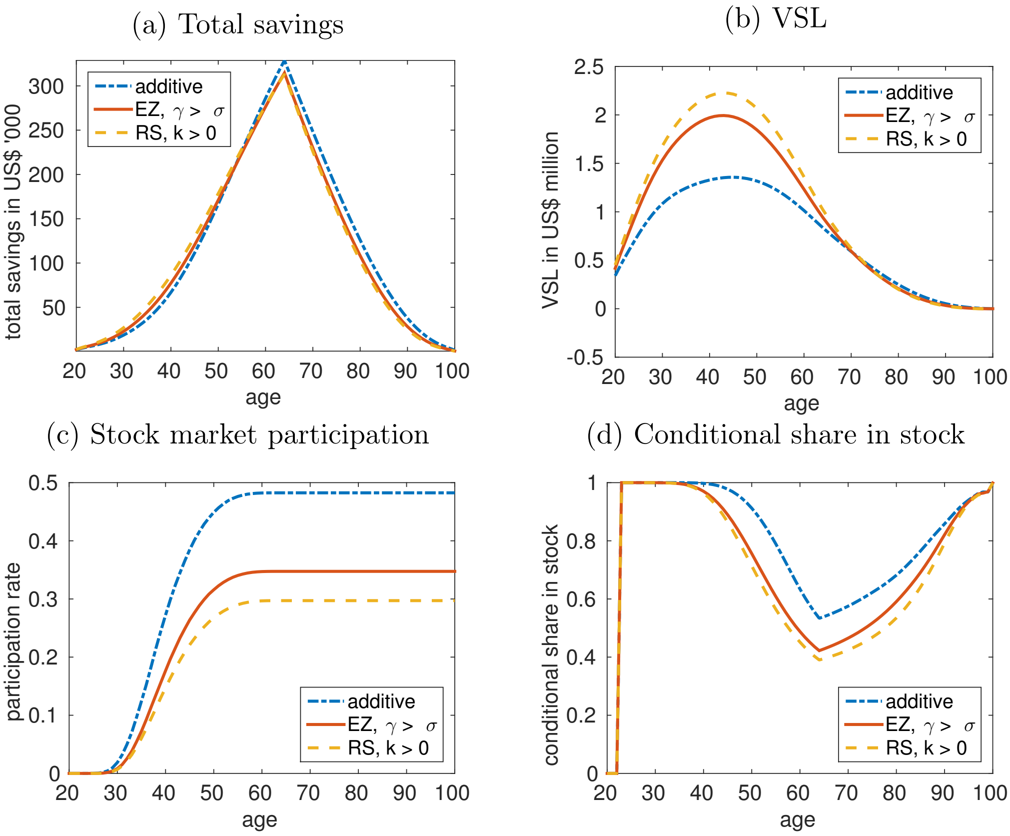


Table 4: Calibrated parameters with no bequest utility

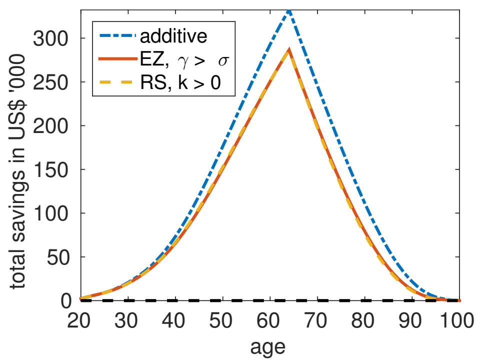
Parameter	Value
Risk aversion, risk-sensitive, k	0.072
Life-death utility gap, $\overline{v}$	21.01
Discount factor, β	0.967
Strength of bequest motive, θ	$0.0^{\dagger}$
Bequest luxury good strength, $\overline{w}$	$0.247^{\dagger}$
Participation cost, F	71.7% of $\bar{y}$

†: The values for θ and $\bar{w}$ are set exogenously here to make the bequest motive ineffective.

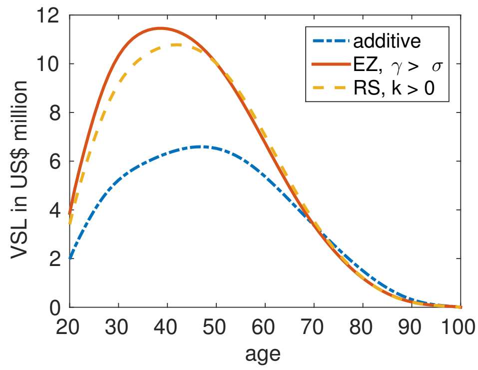
The resulting lifecycle profiles are displayed in Figure 5. Overall, the profiles are very similar to those in the baseline economy. As expected, the main difference are savings at older ages, which now decrease to zero as there is no bequest motive, see panel (a).[28] This stronger fall in assets is the reason why the conditional share in stock, displayed in panel (d), rises more strongly at old age. Nonetheless, our main findings do not rely on the bequest motive: more risk averse agents save less, participate less in the stock market, and—conditional on participating—invest less in stock.

Figure 5: Lifecycle profiles for economy without bequest motive

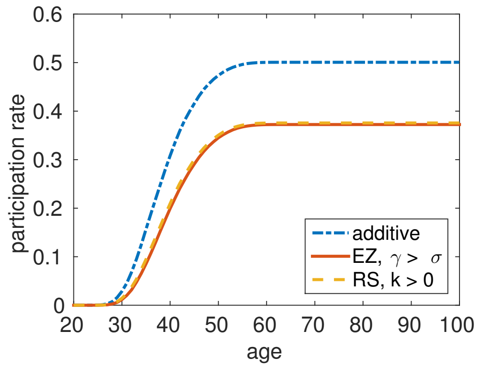
(a) Total savings



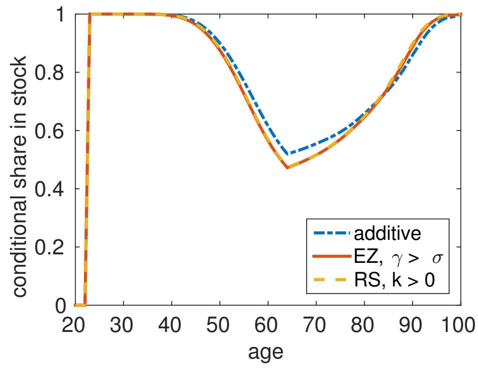
(b) VSL



(c) Stock market participation



(d) Conditional share in stock



C.3 Alternative stock market participation costs

C.3.1 Participation cost proportional to persistent labor component

We assume here that the once-in-a-lifetime stock market participation cost is proportional to the persistent component in labor income, which consists of the per-

[28]There are, of course, still accidental bequests.

sistent stochastic component and the deterministic component, $\{\pi_t + \mu_t\}_t$. This follows Gomes and Michaelides (2005), among others, and can be interpreted as an opportunity cost. More precisely, the agent pays the cost $Fe^{\pi_t+\mu_t}$ (instead of F as in the main text) when she decides to participate in the stock market at date t. The budget constraint (13) when alive becomes:

$$c_t + b_t + s_t + Fe^{\pi_t+\mu_t} 1_{\eta_t=1} 1_{\eta_{t-1}=0} = y_t + R^f b_{t-1} + R_t^s s_{t-1}.$$

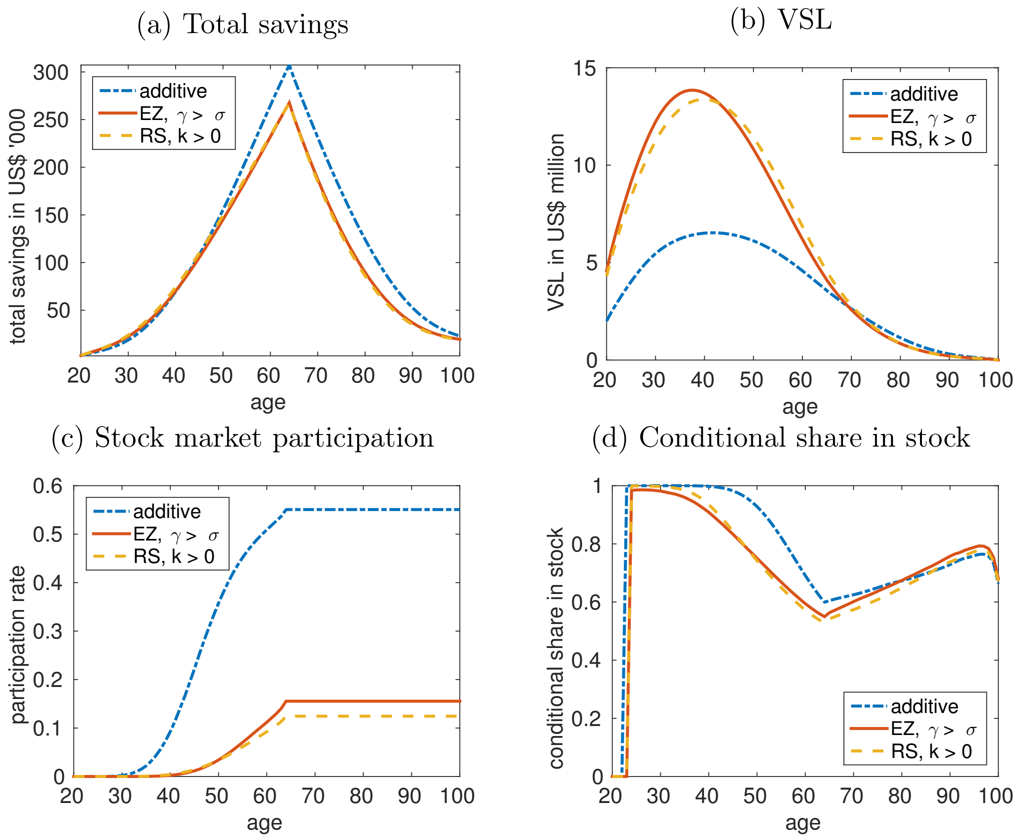
The agent's program is otherwise unchanged. The new values for the calibrated parameters are displayed in Table 5. The main difference to the baseline calibration is the participation cost that drops from 83.2% to 58.3% of average income $\overline{y}$.

Table 5: Calibrated parameters with proportional cost

Parameter	Value
Risk aversion, risk-sensitive, k	0.078
Life-death utility gap, $\overline{v}$	24.85
Discount factor, β	0.958
Strength of bequest motive, θ	11.01
Bequest luxury good strength, $\overline{w}$	0.247
Participation cost, F	58.3% of $\bar{y}$

Results are plotted in Figure 6. Compared to our baseline results, the main difference is that the Epstein-Zin and risk-sensitive agents both participate much less in the stock market, cf. panel (c). The reason is that a sequence of good shocks meant in the baseline that the agent was willing to pay the participation cost, but now she faces the highest participation cost, so that it's more likely she will abstain. By contrast, if the the agent experiences a sequence of bad shocks, the lower participation cost does not immediately induce her to buy stocks, because she first wants to build up a precautionary buffer of safe assets. Other profiles are very similar to our baseline. Therefore, summing up, the results confirm our main findings that more risk averse agents save less, participate less in financial markets, and when they do, purchase a smaller amount of stocks.

Figure 6: Lifecycle profiles for economy with proportional cost



C.3.2 Participation cost every period

We here assume that the stock market participation cost is paid in every period when the agent is trading stocks. This follows Fagereng et al. (2017), among others. The budget constraint (13) when alive becomes:

$$c_t + b_t + s_t + F1_{s_t>0} = y_t + R^f b_{t-1} + R^s_t s_{t-1}.$$

The rest of the agent's program remains unchanged. Note, however, that since the stock market participation is decided in every period, the participation status, denoted by η in the original program, is not needed as a state variable any more. The parameter values are presented in Table 6—all other parameters are the same as in the baseline calibration, cf. Table 1. The main difference to the baseline is the participation cost that now falls from 83.2% to 3.74% of average income, $\overline{y}$. This is comparable with—though still higher than—values reported in the literature (see, for example, Fagereng et al. 2017 and Vissing-Jorgensen 2002).[29]

[29] The participation cost could be pushed further down by assuming a higher risk aversion (like

Table 6: Calibrated parameters with per period cost

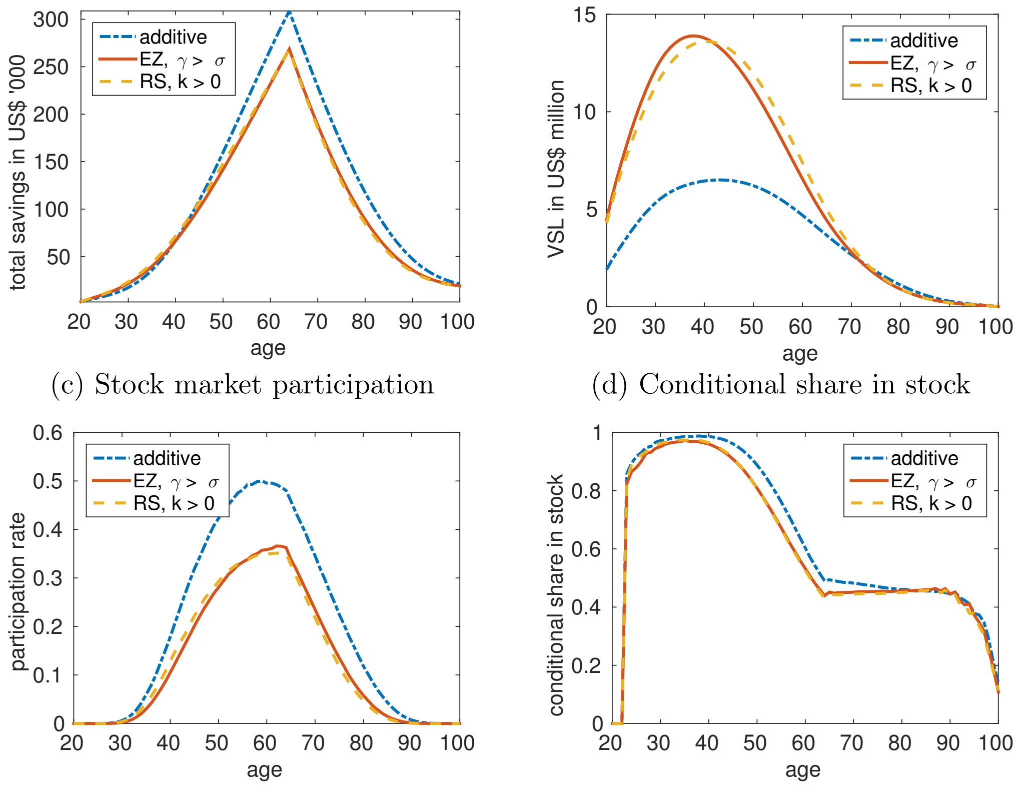
Parameter	Value
Risk aversion, risk-sensitive, k	0.082
Life-death utility gap, $\overline{v}$	24.74
Discount factor, β	0.956
Strength of bequest motive, θ	11.12
Bequest luxury good strength, $\overline{w}$	0.243
Participation cost, F	3.74% of $\bar{y}$

Results are plotted in Figure 7. Results regarding savings and VSL do not change: more risk averse agents save less and have a higher VSL. However, the stock market participation is now hump shaped, and tracks the lifecycle evolution of wealth, cf. panel (c). In older ages, when agents start decumulating their savings, they participate less in financial markets. The participation pattern is then more consistent with the data (see Alan 2006 or Fagereng et al. 2017, for instance). The other important difference to the baseline is that the conditional stock share does not increase after retirement, which is also more consistent with the data. Indeed, for the additive agent, it is monotonously declining after age 40, and nearly monotonously declining for the Epstein-Zin and the risk-sensitive agents. Despite these important differences, our main results are again clearly confirmed: more risk averse agents save less, participate less in the stock market, and—conditionally on participating—invest a smaller share in stock.

in Fagereng et al. 2017), or a participation cost that is both per period and proportional to persistent income.

Figure 7: Lifecycle profiles for economy with per period cost

(a) Total savings (b) VSL (c) Stock market participation (d) Conditional share in stock



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