Optimal fiscal policy with heterogeneous agents and aggregate shocks*

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Abstract

We derive an optimal fiscal policy in a heterogeneous-agent model with capital accumulation and aggregate shocks and where the government uses public debt, and capital and labor taxes, to smooth technology and public spending shocks. In order to perform this analysis, we provide a new representation of incomplete insurance-market economies, based on a truncation theory in the space of idiosyncratic histories. The steady-state capital tax is shown to be positive and to depend on the severity of credit constraints. Labor tax is more volatile with heterogeneous agents compared to a representative agent economy. The optimal public debt is higher in the incomplete-market model and its dynamics are different from its complete-market counterpart. Finally, the difference between the complete and incomplete market economies is quantitatively significant for public spending shocks, less so for technology shocks.

Keywords: Incomplete markets, optimal policy, public debt.

JEL codes: E21, E44, D91, D31.

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1 Introduction

Incomplete insurance-market economies provide a useful framework for examining many relevant aspects of inequalities and individual risk in general equilibrium. In these models, infinitely-lived agents face incomplete insurance markets and borrowing limits that prevent them from perfectly hedging their idiosyncratic risk, in line with the Bewley-Huggett-Aiyagari literature (Bewley 1983, Imrohoroğlu 1989, Huggett 1993, Aiyagari 1994, Krusell and Smith 1998). These frameworks are becoming increasingly popular and are now widely used, since they fill a gap between micro- and macroeconomics and enable the inclusion of aggregate shocks and a number of additional frictions on both the goods and labor markets. However, in terms of normative analysis, little is known about optimal policies in these environments, due to the difficulties generated by the large and time-varying heterogeneity across agents. This is unfortunate, since a vast literature, reviewed below, suggests that the interaction between wealth inequalities and capital accumulation has first-order implications for the optimal design of time-varying fiscal policies.

This paper presents a methodological contribution that offers a general and tractable representation of incomplete insurance-market economies. This representation allows us to easily solve the Ramsey problem in economies with both capital and aggregate shocks. We apply our framework to provide a theoretical and quantitative analysis of optimal fiscal policy. We derive new results about the optimal dynamics of public debt, distorting capital and labor taxes, and transfers, considering rich trade-offs involving redistribution, insurance, and incentives.

Heterogeneity increases with time in incomplete insurance market economies because agents differ according to the full history of their idiosyncratic risk realizations. Huggett (1993), using the results of Hopenhayn and Prescott (1992), and Aiyagari (1994) have shown that economies without aggregate risk have a recursive structure when the distribution of wealth is introduced as a state variable. Unfortunately, the distribution of wealth has infinite support, which is at the root of many analytical difficulties. Our methodological contribution is to represent incomplete insurance-market economies as economies with finite support. More precisely, we construct an environment where agent heterogeneity depends only on a finite but arbitrarily large number, denoted N, of consecutive past realizations of idiosyncratic risk. As a theoretical outcome, agents having the same idiosyncratic risk history for the previous N periods choose the same consumption and wealth levels. The interest of this *truncated* representation of incomplete insurance-market economies stems from four properties. First, the allocation can be represented as a theoretical outcome with partial insurance, which ensures the existence of the equilibrium in the presence of aggregate shocks. Second, the amount of insurance implicitly provided by the truncation converges toward 0, for large N, under general conditions.

Third, and more importantly, as our representation has a finite state space, we can use a Lagrangian approach as in Marcet and Marimon (2011) to derive Ramsey problems. These appear to be powerful tools with which to derive economic intuitions for optimal policies in incomplete insurance-market economies. Finally, the finite state space greatly simplifies the simulation of the model, as standard perturbation methods can be used.

The goal of our paper is to use this framework to analyze how incomplete insurance markets for idiosyncratic risk change our understanding of optimal fiscal policy, both in the steady state and after aggregate shocks. To do so, we derive the optimal fiscal policy in a model with an incomplete insurance market for employment risk ("IM economy", hereafter) and with aggregate risk and compare the results with those generated in a complete insurance-market economy ("CM economy", hereafter). A natural benchmark for this investigation is the economy studied in Chari, Christiano, and Kehoe (1994), where: 1) the government has three instruments with which to finance public spending: public debt and a linear tax on labor and on capital; 2) the government solves a Ramsey problem with full commitment; and 3) both a technology and a public spending shock are considered. Obviously, each of these assumptions has been relaxed in the vast literature surveyed below, and could be introduced in our IM economy. Nevertheless, comparing the IM and CM economies in this benchmark case delivers a number of new results on the specific effect of market incompleteness on optimal fiscal policy. We provide both theoretical and quantitative results, simulating CM and IM economies with standard parameter values. Using the work of Chamley (1986) and Judd (1985a) (for the steady state) and of Chari, Christiano, and Kehoe (1994) (for the dynamics), we can summarize the main results in the CM economy as follows:[1] a) optimal capital taxes are 0 in the interior steady state; b) capital taxes vary considerably during the period of a shock; c) the expected capital tax rate is close to its steady-state value one period after the shock; d) labor taxes remain roughly constant after the shock; and e) public debt varies to smooth taxes.

Market incompleteness changes the previous results as follows. a) We prove that capital taxes are positive at the steady state if, and only if, credit constraints are binding for some agents. We derive an expression for the optimal capital tax, enabling us to discuss previous results in the literature (Aiyagari 1995 among others). We show that market incompleteness is not sufficient to provide a deviation from the Chamley-Judd result of a zero capital tax at the steady state, which requires binding credit constraints. In our quantitative investigation, we find that optimal average capital tax is around 8%. b) and c) As in CM, the capital tax in IM varies in the period of the shock before going back to its steady state value. However, we find that capital tax has a redistributive effect in the IM economy, which limits its use along the business cycle. d) We find that

[1] The optimality of the interior solution has been analyzed in Straub and Werning (2014). We discuss this in our setup in Section 4 below.

labor taxes are more volatile in the IM economy than in the CM economy, although their steady-state values are close to one other. The reason for this is that labor tax generates additional redistributive effects (for which we provide an expression), increasing its use along the business cycle. e) The optimal steady-state level of public debt is negative, but different, in both economies. We find that optimal public debt is higher in the IM economy, by 6 percentage points of GDP. This extra level of public debt is used as liquidity by households to self-insure. In addition, the dynamics of public debt are very different in the IM and CM economies, resulting from the different cyclical properties of taxes. Finally, we quantitatively find that there are large differences between IM and CM economies after a public spending shock (directly affecting the budget constraint of the government) compared to a TFP shock.

Related literature. Our paper is related to two vast streams of literature. The first is the theory and quantitative work on heterogeneous agent models. The second is the literature on optimal fiscal policy.

This paper first provides a new tractable representation of incomplete insurance markets (i.e., a finite dimensional state-space representation). Some environments already provide a tractable framework. This is the case for no-trade equilibria with permanent idiosyncratic shocks (Constantinides and Duffie 1996), used for instance in Heathcote, Storesletten, and Violante (2017). Krusell, Mukoyama, and Smith (2011) study a class of no-trade equilibria in an economy without capital and with a tight-enough credit constraint, used in Ravn and Sterk (2017). Departing from no-trade equilibria, a class of "small trade" equilibria, featuring "reduced heterogeneity" with a finite number of wealth levels, has been studied (Challe and Ragot 2016, LeGrand and Ragot 2016, Challe, Matheron, Ragot, and Rubio-Ramirez 2017, Ragot 2018). The current paper extends these previous works and provides a general theory of truncated representations of incomplete-market economies, which are a consistent representation of Bewley economies. In addition, it derives new tools for studying optimal policies, based on the dynamic structure of Lagrange multipliers.

Second, our paper contributes to the literature on distortions and optimal policies in incomplete insurance-market models. Some papers have studied the effects of given fiscal experiments in a heterogeneous agent framework, such as Heathcote (2005), who considers aggregate shocks, and Kaplan and Violante (2014), who consider a fiscal transfer. Heathcote and Perri (2017) analyze equilibrium multiplicity in an economy without capital. For the design of optimal policies, many contributions identify a number of relevant trade-offs, but to the best of our knowledge, the general case with capital accumulation and aggregate shocks has not yet been studied. In economies without aggregate shocks, Aiyagari (1995) shows that the steady-state capital tax can be non-negative. Aiyagari and McGrattan (1998) compute the optimal steady-state level of debt. Dávila, Hong, Krusell, and Ríos-Rull (2012) show that the steady-state capital stock can be too low, solving for a constrained efficient allocation. Açikgöz (2015) solves a Ramsey problem to obtain the steady-state fiscal policy and level of public debt. Gottardi, Kajii, and Nakajima (2015) compute the Ramsey allocation in a model with human capital accumulation. Nuño and Moll (2017) use a continuous-time approach and mean-field games to characterize differences in inequalities for economies without aggregate shocks. Shin (2006) studies a two-agent economy to derive additional results. Recently, Bhandari, Evans, Golosov, and Sargent (2013, 2017b) have derived results about optimal policies in environments with incomplete insurance markets and aggregate shocks. They study an economy without capital, with lump-sum taxes, and where credit constraints are loose enough such that they never bind in equilibrium. They show that public debt is irrelevant, which simplifies the state space and allows for the introduction of additional features, such as nominal frictions. Instead, we study an economy with capital (and capital tax) and we allow for binding credit constraints.

Third, this paper is also related to the vast literature on optimal fiscal policy with aggregate shocks. Seminal contributions consider a complete-market economy with a representative agent (Barro 1979, Lucas and Stokey 1983, surveyed in Chari and Kehoe 1999). More recent contributions consider incomplete markets for the aggregate risk (but with complete insurance markets), introducing non state-contingent public debt (Aiyagari, Marcet, Sargent, and Seppälä 2002, Farhi 2010, Bhandari, Evans, Golosov, and Sargent 2017a). Several papers have additionally introduced ex-ante agent heterogeneity (see Bassetto 2014, Azzimonti, de Francisco, and Krusell 2008a and 2008b, Azzimonti and Yared 2017, Correia 2010, Greulich, Laczó, and Marcet 2016). The New Dynamic Public Finance literature focuses on optimal fiscal policy in environments with heterogeneous and private information (Mirrlees 1971, Golosov and Tsyvinski 2007, Werning 2007 among others). Here, we focus on a Ramsey approach, limiting the number of instruments, which helps to identify some basic tradeoffs (see Farhi and Werning 2013, and Golosov, Tsyvinski, and Werquin 2016 for a discussion).

This paper is also, but more indirectly, related to the computational literature studying incomplete insurance markets with perturbation methods. Reiter (2009) uses perturbation methods to solve for aggregate dynamics, by discretizing the wealth distribution to obtain a finite-dimensional state space.[2] Other authors have used truncation strategy for aggregate shocks as an approximation device in computational procedures. Instead, we construct economies that deliver a finite-dimensional state space in the space of idiosyncratic histories, as a *theoretical* outcome. This last property is key to deriving optimal Ramsey policies.

[2]Other numerical methods using perturbation methods are developed in Mertens and Judd (2018), Preston and Roca (2007), Kim, Kollmann, and Kim (2010), or Winberry (2018), who approximates the wealth distribution by a finite number of parameters.

The rest of the paper is organized as follows. In Section 2, we present the environment. We describe the family head problem and derive the associated allocation in Section 3. We then show the decentralization mechanism in Section 3.2. We solve the Ramsey problem in Section 4, where we introduce the complete-market economy. In Section 5, we provide a numerical application illustrating our findings. Finally, conclusions are given in Section 6.

2 The environment

Time is discrete, indexed by $t \geq 0$. The economy is populated by a continuum of agents of size 1, distributed on a segment J following a non-atomic measure ℓ: $\ell(J) = 1$.[3]

2.1 Risk

Aggregate risk. The aggregate risk is represented by a probability space $(\mathcal{S}^\infty, \mathcal{F}, \mathbb{P})$. In any period t, the aggregate state, denoted s_t, takes values in the state space $\mathcal{S} \subset \mathbb{R}^+$ and follows a first-order Markov process. The history of aggregate shocks up to time t is denoted by $s^t = \{s_0, \ldots, s_t\} \in \mathcal{S}^{t+1}$. Finally, the period-0 probability density function of any history s^t is denoted by $m^t(s^t)$.

Idiosyncratic risk. At the beginning of each period, agents face an uninsurable idiosyncratic labor productivity shock e_t that can take $E+1$ values in the set $\mathcal{E} = \{0, \ldots, E\} \in \mathbb{R}_+^{E+1}$. Agents in state $e \in \mathcal{E}$, $e \neq 0$, have a labor productivity $\theta_e > 0$, which is assumed to be increasing in e, without loss of generality. Agents in state $e = 0$ have a zero market productivity, i.e. $\theta_0 = 0$, but devote a fixed amount of $\delta > 0$ labor units to earn a home production of δ units of final goods. The first type of agents can be considered as employed workers with various productivities, while the latter can be considered as unemployed workers. This modeling choice enables us to cover the various cases that can be found in the literature.

The productivity shock e_t follows a discrete irreducible and aperiodic first-order Markov process with transition matrix $M(s_t) \in [0,1]^{(E+1)\times(E+1)}$. The probability $M_{e,e'}(s_t)$ is the probability that an agent switches from state e at date t to state e' at date $t+1$, when the aggregate state is s_t in period t. The history of idiosyncratic shocks up to date t is denoted by $e^t = \{e_0, \ldots, e_t\} \in \mathcal{E}^{t+1}$.

Remark 1 (Notations) *For the sake of clarity, we will denote the realization in state s^t of any random variable $X_t : \mathcal{S}^t \to \mathbb{R}$ by X_t, instead of $X_t(s^t)$, and will denote the realization in state (s^t, e^t) of any random variable $Y_t : \mathcal{S}^t \times \mathcal{E}^t \to \mathbb{R}$, by Y_{t,e^t}.*

[3] We assume that the law of large numbers holds. See Green (1994) for a complete construction of J and ℓ. See also Feldman and Gilles (1985), Judd (1985b), and Uhlig (1996) for other solutions.

2.2 Preferences

In each period, the economy has two goods: a consumption-capital good and labor. Agents rank consumption c and labor l according to a smooth period utility function $U(c,l)$, satisfying standard regularity properties. As is standard in this class of models, we consider a Greenwood-Hercowitz-Huffman (GHH) utility function, exhibiting no wealth effect for the labor supply:[4]

$$U(c,l) = u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right), \tag{1}$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously derivable, increasing, and concave, with $u'(0) = \infty$.

We further assume that the utility agents derive in each period can depend on their idiosyncratic history. In each period t, the period utility of an agent with idiosyncratic history e^t is assumed to be $\xi_{e^t}U(c,l)$, where $\xi_{e^t} : \mathcal{E}^{t+1} \to \mathbb{R}_{++}$ can be interpreted as a taste shock. Introducing taste shocks offers further flexibility, which is only used in the quantitative exercise (Section 5). The model and results we present below are obviously still valid in the standard case where $\xi_{e^t} = 1$ for all e^t.

Each agent ranks consumption and labor streams, denoted respectively by $(c_t)_{t\geq 0}$ and $(l_t)_{t\geq 0}$, according to the intertemporal criterion $\sum_{t=0}^{\infty}\beta^t\xi_{e^t}U(c_t,l_t)$, where $\beta \in (0,1)$ is the discount factor.

2.3 Production and assets

In any period t, a production technology with constant returns to scale (CRS) transforms capital K_{t-1} and labor L_t into $F(K_{t-1}, L_t, s_t)$ units of output. The production function is smooth in K and L and satisfies the standard Inada conditions. Capital must be installed one period before production, and the state of the world may potentially affect productivity through a technology shock. This formulation allows for capital depreciation, which is subsumed by the production function F, as in Farhi (2010) for instance. Labor L_t is measured in efficient units, and is equal to the sum of the individual labor efforts expressed in efficient units: $L_t = \int_{i\in J}\theta_{e_t^i}l_{i,t}\ell(di)$. The good is produced by a unique profit-maximizing representative firm. We denote by $\tilde{w}_t$ the real before-tax wage rate in period t and by $\tilde{r}_t$ the real before-tax rental rate of capital in period t. Profit maximization

[4] All our results can be derived with a general utility function $U(c,l)$. A GHH utility function slightly simplifies the algebra, especially when deriving the Ramsey problem in Section 4. Admittedly, and as shown by Marcet, Obiols-Homs, and Weil (2007), considering an alternative utility function would affect the optimal tax schedule, as aggregate labor supply would depend on wealth distribution.

yields in each period $t \geq 1$ are:[5]

$$\tilde{r}_t = F_K(K_{t-1}, L_t, s_t) \quad \text{and} \quad \tilde{w}_t = F_L(K_{t-1}, L_t, s_t). \tag{2}$$

Finally, agents save buying claims on the capital stock and public debt, which will be substitutes in equilibrium. Agents face borrowing constraints, and their asset holdings must be higher than $-\bar{a} \leq 0$. For important reasons discussed in Section 4, we focus on the case where the credit limit is above the steady-state natural borrowing limit.[6] In addition, some proofs below require us to assume that agents cannot save more than a^{max}. This maximal amount can be chosen to be arbitrarily large, in particular so that it is never a binding constraint.[7]

2.4 Government, fiscal tools, and resource constraints

In each period t, the government has to finance an exogenous and possibly stochastic public goods expenditure $G_t \equiv G_t(s_t)$. The government can levy distorting taxes on capital income τ_t^K or on labor income τ_t^L, or issue an amount B_t of a riskless one-period public bond.[8] As in Heathcote (2005), we assume that the public debt pays the economy-wide interest rate $\tilde{r}_t$ for any aggregate history $s^t \in \mathcal{S}^t$. The same tax rate τ_t^K applies to public bonds and capital shares. Consequently, both assets are perfect substitutes for agents, which avoids the need to consider a portfolio choice.

As is standard, we also assume that the date-0 capital tax rate, borne by initial capital, is set exogenously. Taxing capital in the first period is non-distorting, and the government would heavily tax the initial capital stock (see Sargent and Ljungqvist 2014, Section 16.7 for a discussion). The date-t budget constraint of the government is:

$$G_t + (1 + \tilde{r}_t)B_{t-1} \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \tilde{r}_t A_{t-1} + B_t. \tag{3}$$

We denote the after-tax real interest and wage rates by:

$$r_t = (1 - \tau_t^K)\tilde{r}_t \quad \text{and} \quad w_t = (1 - \tau_t^L)\tilde{w}_t. \tag{4}$$

[5] To simplify the notation, r and w denote the *after-tax* wage and interest rates, as in Aguiar and Amador (2016) among others.

[6] See Aiyagari (1994) for a discussion of the relevant values of $\bar{a}$, called the natural borrowing limit in an economy without aggregate shocks. See Shin (2006) for a discussion including aggregate shocks. A standard value in the literature is $\bar{a} = 0$, which ensures that consumption is positive in all states of the world.

[7] As for instance in Szeidl (2013), the assumption of the maximal bound a^{max} enables us to consider a general utility function. An alternative option would be to assume a bounded periodic utility function u, as in Miao (2006).

[8] The question of the optimal mix of these financing tools will be the focus of the second part of the paper and in particular of the Ramsey problem studied in Section 4.

Using the CRS property of the production function, the budget constraint (3) becomes:

$$G_t + r_t K_{t-1} + w_t L_t + (1+r_t)B_{t-1} \leq F(K_{t-1}, L_t, s_t) + B_t. \tag{5}$$

Finally, if C_t^{tot} denotes the total consumption in period t, the economy-wide resource constraint is $G_t + C_t^{tot} + K_t \leq F(K_{t-1}, L_t, s_t) + K_{t-1} + S_{t,0}(s^t)\delta$, where $S_{t,0}(s^t)$ denotes the size of the population in state $e = 0$ at date t, thus producing δ. For the sake of clarity, we present the formal definition of the equilibrium in the next section.

3 The truncated economy

In general, the previous economy features growing heterogeneity in wealth levels over time, because agents with different idiosyncratic histories will choose different savings. This heterogeneity can be represented by a time-varying distribution of wealth levels with infinite support, which raises considerable theoretical and computational challenges. We now present an environment in which the savings of each agent depend on the idiosyncratic risk realizations for a given number of consecutive past periods, rather than on the whole history. As an endogenous outcome, the heterogeneity among the population is summarized by a finite (but possibly large) number of agent types.

To simplify the exposition, we present this economy in three steps. First, we use the family and island metaphor (see Lucas 1975 and 1990, or Heathcote, Storesletten, and Violante 2017 for a more recent reference) as a direct constraint on the environment. We then derive a recursive decentralization. Finally, we present the risk-sharing arrangement across households, which generates the given truncation. The advantage of this presentation strategy is that the existence of an equilibrium can be proved using standard techniques.

We denote by $N \geq 0$ the length of the truncation for idiosyncratic histories. We assume that taste shocks ξ_{e^t} depend only on the past N periods' shocks in period t, to be consistent with the truncation structure.

3.1 The island metaphor

Island description. There are $(E+1)^N$ different islands, where the cardinal of the set $\mathcal{E}$ of idiosyncratic risk realizations is $E+1$. Agents with the same idiosyncratic history for the last N periods are located on the same island. Any island is represented by a vector $e^N = (e^N_{-N+1}, \ldots, e^N_0) \in \mathcal{E}^N$ summarizing the N-period idiosyncratic history of all island inhabitants. At the beginning of each period, agents face a new idiosyncratic shock. Agents with history $\hat{e}^N \in \mathcal{E}^N$ in the previous period are endowed with history e^Nin the current period, and we denote $e^N \succeq \hat{e}^N$ when e^N is a possible continuation of $\hat{e}^N$. The specification $N = 0$ corresponds to the full insurance case (only one island),

and thus to the standard representative-agent assumption, which will be used below as a benchmark. Symmetrically, the case $N = \infty$ corresponds to a standard incomplete-market economy with aggregate shocks, in line with Krusell and Smith (1998). To simplify the exposition, we assume that all agents enter the economy with identical initial wealth $(a_{-1,e^N})_{e^N \in \mathcal{E}^N} = a_0$.

The family head. The family head maximizes the welfare of the whole family on all islands, attributing an identical weight to all agents and behaving as a price-taker.[9] The family head can freely transfer resources among agents on the same island, but cannot do so across islands. All agents belonging to the same island are treated identically and will therefore receive the same allocation, as is consistent with welfare maximization. For agents on any island $e^N \in \mathcal{E}^N$, the family head will choose the per capita consumption level c_{t,e^N}, the labor supply l_{t,e^N}, and the end-of-period savings a_{t,e^N} (remember that capital and public debt are substitutes).

Island sizes. The probability $\Pi_{t,\hat{e}^N,e^N}$ that an agent with history $\hat{e}^N = (\hat{e}^N_{-N+1}, \ldots, \hat{e}^N_0)$ in period t experiences history $e^N = (e^N_{-N+1}, \ldots, e^N_0)$ in period $t+1$ is the probability of switching from state $\hat{e}^N_0$ at t to state e^N_0 at $t+1$, provided that histories $\hat{e}^N$ and e^N are compatible. Formally, we have $\Pi_{t,\hat{e}^N,e^N} = 1_{e^N \succeq \hat{e}^N} M_{\hat{e}^N_0, e^N_0}(s_t)$, where $1_{e^N \succeq \hat{e}^N} = 1$ if e^N is a possible continuation of history $\hat{e}^N$ and 0 otherwise. We can thus deduce the law of motion of island sizes $(S_{t,e^N})_{t \geq 0, e^N \in \mathcal{E}^N}$:

$$S_{t+1,e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} S_{t,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N}, \tag{6}$$

where the initial size of each island $(S_{-1,e^N})_{e^N \in \mathcal{E}^N}$, with $\sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} = 1$, is given. The law of motion (6) is thus valid from period 0 onwards.

Timing. At the beginning of each period t, agents learn their current idiosyncratic shock and have to move from island $\hat{e}^N$ to island e^N. The family head cannot change the allocation before agents leave the island. As a consequence, agents move by taking their wealth with them, equal to the per capita saving $a_{t-1,\hat{e}^N}$. On island e^N, the wealth of all agents coming from island $\hat{e}^N$ (equal to $S_{t-1,\hat{e}^N} \Pi_{t-1,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}$) – and for all islands $\hat{e}^N$– is pooled together and then equally divided among the S_{t,e^N} agents of island e^N. Therefore, at the beginning of period t, each agent holds wealth $\tilde{a}_{t,e^N}$ equal to:

$$\tilde{a}_{t,e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t-1,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}. \tag{7}$$

[9] As the family head does not internalize the effect of its choice on prices, the allocation is not constrained-efficient, and the distortions identified by Davila et al. (2012) are present in the equilibrium allocation. The planner will reduce them with its instruments, defined in Section 4.

The program of the family head can now be expressed as follows:[10]

$$\max_{\left(a_{t,e^N}, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0, e^N\in\mathcal{E}^N}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{e^N\in\mathcal{E}^N} S_{t,e^N}\xi_{e^N} U\left(c_{t,e^N}, l_{t,e^N}\right)\right], \tag{8}$$

$$a_{t,e^N} + c_{t,e^N} = w_t\theta_{e_0^N} l_{t,e^N} + \delta 1_{e_0^N=0} + (1+r_t)\tilde{a}_{t,e^N}, \text{ for all } e^N \in \mathcal{E}^N, \tag{9}$$

$$c_{t,e^N},\ l_{t,e^N} \geq 0, a_{t,e^N} \geq -\bar{a}, \text{ for all } e^N \in \mathcal{E}^N, \tag{10}$$

$$(S_{-1,e^N})_{e^N\in\mathcal{E}^N} \text{ and } a_0 \text{ are given,} \tag{11}$$

and subject to $l_{t,e^N} = \delta$ if $e_0^N = 0$, to the law of motion (6) for $(S_{t,e^N})_{t\geq 0}^{e^N\in\mathcal{E}^N}$, and to the definition (7) of $(\tilde{a}_{t,e^N})_{t\geq 0}^{e^N\in\mathcal{E}^N}$.[11]

The family head maximizes aggregate welfare (8) subject to the budget constraints (9) on all islands, to positivity and borrowing constraints (10), and to initial conditions (11). As the objective function is increasing and concave, constraints are linear (i.e., the admissible set is convex), and allocations are bounded (a^{max} guarantees a compact admissible set), the existence of the equilibrium can be proved using standard techniques (see Stokey and Lucas (1989, Chap. 15 and 16)). We therefore omit this proof in the interest of conciseness.[12] If $\beta^t\nu_{t,e^N}m(s^t)$ denotes the Lagrange multiplier of the credit constraint of island e^N, the first-order conditions are:

$$\xi_{e^N}U_c(c_{t,e^N}, l_{t,e^N}) = \beta\mathbb{E}_t\left[\sum_{\tilde{e}^N\succeq e^N}\Pi_{t,e^N,\tilde{e}^N}\xi_{\tilde{e}^N}U_c(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N})(1+r_{t+1})\right] + \nu_{t,e^N}, \tag{12}$$

$$l_{t,e^N} = \left(w_t\theta_{e_0^N}\right)^{\varphi} + \delta 1_{e_0^N=0}, \tag{13}$$

$$\nu_{t,e^N}(a_{t,e^N} + \bar{a}) = 0 \text{ and } \nu_{t,e^N} \geq 0. \tag{14}$$

To anticipate Section 3.2 below, the first-order conditions (12)–(14) have the same form as those derived in standard incomplete insurance-market models. Although the family head cares about agents moving across islands, the result is similar to that of individuals self-insuring against income risk, due to the law of large numbers.

Labor market. On any island e^N, the market labor supply in efficient units at date t amounts to $\theta_{e_0^N}S_{t,e^N}l_{t,e^N}$ (we recall that $\theta_0 = 0$). Summing across all islands yields the total labor supply:

$$L_t = \sum_{e^N\in\mathcal{E}^N}\theta_{e_0^N}S_{t,e^N}l_{t,e^N}. \tag{15}$$

[10]We denote by e_0^N the current idiosyncratic state in island e^N, and $1_{e_0^N=0}$ equals 1 if $e_0^N = 0$ and 0 otherwise.

[11]Note that $\mathbb{E}_t[\cdot]$ in (8) is the expectation operator at date $t \geq 0$ over all future aggregate histories.

[12]Due to the finite heterogeneity representation, we could also prove the existence of a recursive equilibrium. In the interest of conciseness, we do not present this recursive formulation, as it is not necessary for deriving first-order conditions.

Financial market. Total end-of-period savings of all agents, denoted by A_t at date t are:

$$A_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} a_{t,e^N} = \sum_{e^N \in \mathcal{E}^N} S_{t+1,e^N} \tilde{a}_{t+1,e^N}, \tag{16}$$

where the last equality stems from the pooling equation (7). The clearing of the financial market at date t implies that at any date t, the following equality holds:

$$A_t = B_t + K_t. \tag{17}$$

We can now state our sequential equilibrium definition, which is similar to Aiyagari, Marcet, Sargent, and Seppälä (2002) and Farhi (2010).

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations* $\left(c_{t,e^N}, l_{t,e^N}, \tilde{a}_{t,e^N}, a_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$*, of island population sizes* $\left(S_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$*, of aggregate quantities* $(L_t, A_t, B_t, K_t)_{t\geq 0}$*, of price processes* $(w_t, r_t, \tilde{r}_t, \tilde{w}_t)_{t\geq 0}$*, and of a fiscal policy* $(\tau^K_{t+1}, \tau^L_t, B_t)_{t\geq 0}$*, such that, for an initial distribution of island population and wealth* $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N \in \mathcal{E}^N}$*, and for initial values of capital stock* $K_{-1} = \sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} a_{-1,e^N}$*, of public debt* B_{-1}*, of capital tax* τ_0*, and of the initial aggregate shock* s_{-1}*, we have:*

1. *given prices, individual strategies* $\left(a_{t,e^N}, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$ *solve the agents' optimization program in equations (8)–(11);*

2. *island sizes and beginning-of-period individual wealth* $\left(S_{t,e^N}, \tilde{a}_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$ *are consistent with the laws of motion (6) and (7);*

3. *labor and financial markets clear at all dates: for any* $t \geq 0$*, equations (15)–(17) hold;*

4. *the government budget constraint (5) holds at any date;*

5. *factor prices* $(w_t, r_t, \tilde{r}_t, \tilde{w}_t)_{t\geq 0}$ *are consistent with (2) and (4).*

The equilibrium has a simple structure defined at each date by $6(E+1)^N + 8$ variables and $6(E+1)^N + 8$ equations for a given fiscal policy $(\tau^K_{t+1}, \tau^L_t, B_t)_{t\geq 0}$, which is endogenized below.

3.2 Decentralization and convergence properties

We now prove that the previous program can be decentralized through fiscal transfers, which are shown to measure the degree of idiosyncratic risk sharing achieved by asset pooling in the island economy. We prove that these transfers – as well as idiosyncratic risk sharing – converge toward zero for large N, under general conditions. We start with given factor prices and without aggregate shocks, and then introduce aggregate shocks below. First, dropping aggregate shocks implies that we have existence proof of a recursive representation (see Huggett 1993). Second, fixing factor prices avoids potential issues relating to equilibrium multiplicity, as shown in Açikgöz (2016) for instance.

The economy is now similar to that in Section 2, except for the following differences. First, we consider as given a constant after-tax interest rate r – with $\beta(1+r) < 1$ – and an after-tax wage w. Second, no family head imposes allocations, and agents are expected-utility maximizers taking fiscal policy as given. Finally, at each date each agent receives a lump-sum transfer $\Gamma_{N+1}(e^{N+1})$, which is contingent on her individual history e^{N+1} over the previous $N+1$ periods. This is the actual difference compared to a standard incomplete-market framework. Using standard techniques, the agents' program can be written recursively as:[13]

$$V_{N+1}(a, e^{N+1}) = \max_{a',c,l} \xi_{e^N} U(c,l) + \beta \mathbb{E}\left[\sum_{(e^{N+1})' \succeq e^{N+1}} \Pi_{e^{N+1},(e^{N+1})'} V_{N+1}(a', \left(e^{N+1}\right)')\right], \quad (18)$$

$$a' + c = w\theta_{e_0^N} l + \delta 1_{e_0^N=0} + (1+r)a + \Gamma_{N+1}(e^{N+1}), \quad (19)$$

$$c,\, l \geq 0,\, a' \geq -\bar{a}, \quad (20)$$

with $l = \delta$ if $e_0^N = 0$, and where $V_{N+1} : [-\overline{a}, a^{max}] \times \mathcal{E}^{N+1} \to \mathbb{R}$ is the value function, and e_0^N the current idiosyncratic shock realization. With a slight abuse of notation, Π denotes here the transition matrix for $N+1$ histories derived from the matrix M for idiosyncratic states. Compared to the economies studied by Huggett (1993) and Aiyagari (1994), the individual history e^{N+1} is a state variable, as it determines the transfer $\Gamma_{N+1}(e^{N+1})$. The Lagrange multiplier of the credit constraint $a' \geq -\bar{a}$ is denoted ν, and the solution of this program comprises the policy rules g_c^{N+1}, $g_{a'}^{N+1}$, g_l^{N+1}, and g_ν^{N+1} – defined over $[-\overline{a}, a^{max}] \times \mathcal{E}^{N+1}$ – determining consumption, savings, labor supply, and the Lagrange multiplier of the individual budget constraint, respectively. We now propose our characterization result, which states that we can find a particular set of transfers – denoted by $(\Gamma^*_{N+1}(e^{N+1}))_{e^{N+1}\in\mathcal{E}^{N+1}}$ – such that the decentralized economy allocations match those of the family head economy.

Proposition 1 (Finite state space) *There exists a set of balanced transfers, denoted by $(\Gamma^*_{N+1}(e^{N+1}))_{e^{N+1}\in\mathcal{E}^{N+1}}$, such that any optimal allocation of the family head program (8)–(11) is also a solution to the decentralized program (18)–(20).*

The previous proposition states that the family head program presented in Section 3 can be decentralized by the balanced lump-sum transfers $(\Gamma^*_{N+1}(e^{N+1}))_{e^{N+1}\in\mathcal{E}^{N+1}}$ (shortened to Γ^*_{N+1} henceforth). This transfer is formally given in Appendix A, equation (46). It

[13]In line with the literature, we denote the savings choice in the current period by a'. The value a is thus the beginning-of-period wealth.

involves pooling the resources of all agents with the same idiosyncratic history for $N+1$ periods, and redistributing the same amount to agents with the same idiosyncratic history for N periods, such that there are only $(E+1)^N$ possible wealth levels. Thus, the transfers Γ^*_{N+1} mimic the wealth pooling of the island economy, formalized in equation (7), that occurs when agents transfer from one island to another.

As stated in the following proposition, these transfers can be shown to converge to zero for large N.

Proposition 2 (Convergence) *For given factor prices and for a set of transfers equal to* Γ^*_{N+1}*, if* $\kappa \in (0,1)$ *and* $\overline{N} \geq 1$ *exist, such that for all* $N \geq \overline{N}$*, such that for all* $(e_{\bar{N}-1}, \ldots, e_0) \in \mathcal{E}^{\bar{N}}$*, and* $(f_N, \ldots, f_{\bar{N}}), (g_N, \ldots, g_{\bar{N}}) \in \mathcal{E}^{N-\bar{N}+1}$ *and* $a_1, a_2 \in [-\overline{a}, a^{max}]$*:*

$$\left| g_{a'}^{N+1}(a_1, (f_N, \ldots, f_{\bar{N}}, e_{\bar{N}-1}, \ldots, e_0)) - g_{a'}^{N+1}(a_2, (g_N, \ldots, g_{\bar{N}}, e_{\bar{N}-1}, \ldots, e_0)) \right| < \kappa \left| a_1 - a_2 \right|, \tag{21}$$

then:

$$\lim_{N \to \infty} \sup_{e^{N+1} \in \mathcal{E}^{N+1}} \left| \Gamma^*_{N+1}(e^{N+1}) \right| = 0.$$

Although it may appear complicated, condition (21) has a simple meaning. It states that the marginal propensity to save is always smaller than 1 for all agents, as soon as N is high enough. When this condition is fulfilled, transfers tend toward 0 as the length of idiosyncratic history N increases. If the saving propensity is strictly lower than one, initial differences in wealth vanish and agents experiencing the same history of idiosyncratic shocks end up having the same wealth over time. As a consequence, the wealth pooling generated by the transfers Γ^*_{N+1} concerns wealth levels that tend to be closer to one other, and the transfers Γ^*_{N+1} tend toward 0.

Although condition (21) involves the savings policy function rather than model exogenous parameters, the condition can be shown to hold for a large subclass of HARA utility functions. More precisely, for any HARA utility function of the form $c \mapsto \frac{(c+b)^{1-\sigma}}{1-\sigma}$ (or $c \mapsto \ln(c+b)$), with $c > -b$, $0 < \sigma \neq 1$, we can prove that condition (21) holds with $\kappa = (\beta(1+r))^{\frac{1}{\sigma}} \in (0,1)$.[14] Indeed, following the same steps as Açikgöz (2016), we can show that the savings policy function is a contraction.[15]

Because of the mapping between the island economy and the decentralized one, the absolute size of transfers Γ^*_{N+1} can be thought of as a measure of the insurance provided by the wealth pooling operation during island transfers. As a result, when the length of history becomes infinitely large, the insurance provided by pooling vanishes for long history lengths, as is the case in standard incomplete market economies. In the quantitative

[14] A more general expression for HARA utility functions is $\frac{(\frac{c}{\sigma}+b)^{1-\sigma}}{1-\sigma}$, which includes the CARA utility functions as a limit case when $\sigma \to \infty$. In this case, the proof does not hold.

[15] This is specifically the proof of Açikgöz (2016)'s Proposition 7. The proof relies first on a lemma (Lemma H.1) that can be extended to HARA utility functions. The second part of the proof is based on a result of Jensen (2017) stating that saving policy functions are convex, which also holds for HARA utility functions.

part of our article, set out in Section 5, we further analyze the convergence properties, in particular the question of a "reasonable" value for N, the impact of aggregate shocks, and the role of the taste parameters $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$.

A risk-sharing arrangement. We have achieved decentralization through a set of fiscal transfers Γ^*_{N+1}, providing additional insights into the insurance provided by truncation. This truncation is the outcome of a decentralized equilibrium. Assume that 1) agents have full commitment at period 0; 2) they are ex ante identical (and thus all agree on a risk-reducing mechanism); and 3) they can enter risk-sharing agreements at each period $t \geq N$ with other agents having the same history between period $t-N$ and period t. At each period $t \geq N+1$ agents with the same history for the last N periods insure each other against heterogeneity in idiosyncratic risks prior to period $t-N$. As an equilibrium outcome, agents insure themselves against the heterogeneous realization of the risk at period $t-N-1$ (because risk at periods $t-N-2$ and before has already been insured in period $t-1$). $\Gamma^*_{N+1}(e^{N+1})$ is then the amount received by an agent with history e^{N+1} from agents with history e^N. As agents are identical in period 0, they all agree to commit to this risk-sharing arrangement, which provides additional but limited insurance. This risk-sharing arrangement mimics the fiscal transfer Γ^*_{N+1}, which in turn mimics the island structure. We do not attempt to provide a microfoundation for this risk-sharing arrangement based on deeper informational frictions (such as the ability to observe agents' past idiosyncratic statuses). In line with the Bewley tradition, we simply use this insurance structure, which provides an intermediate level of insurance between the complete and incomplete insurance-market models, to derive new results about optimal fiscal policies.

4 Optimal fiscal policy

4.1 The Ramsey problem

We now solve the Ramsey problem in our incomplete-market island economy with aggregate shocks. The Ramsey problem requires the government to choose a fiscal policy that maximizes aggregate welfare. This aggregate welfare, computed using a utilitarian criterion, is simply the objective of the family head in equation (8).[16] The following definition formalizes this problem, using the notations of Section 3.

Definition 2 (Ramsey problem for a truncated economy) *Let $N > 0$. Given initial conditions regarding the wealth distribution $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N \in \mathcal{E}^N}$, the initial public debt B_{-1}, the initial capital tax τ_0^K, and the initial aggregate state s_{-1}, the Ramsey problem consists in choosing, at date 0, a fiscal policy comprising capital and labor tax paths*

[16] Alternative social welfare functions can be used, but we focus on the most standard case.

$(\tau_{t+1}^K, \tau_t^L)_{t\geq 0}$, and public debt paths $(B_t)_{t\geq 0}$, that maximizes the aggregate welfare defined in (8) among the set of competitive equilibria characterized in Definition 1.

Since the period-0 capital tax rate is given, the capital tax path starts at date 1. Equation (4) implies that the government is able to set the post-tax interest rate $(r_t)_{t\geq 1}$ and the post-tax wage rate $(w_t)_{t\geq 0}$ instead of the distorting taxes $(\tau_t^K)_{t\geq 1}$ and $(\tau_t^L)_{t\geq 0}$, as in Chamley (1986). As a consequence, we can formalize the Ramsey problem as follows:

$$\max_{\left(r_{t+1},w_t,B_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \sum_{e^N\in\mathcal{E}^N} S_{t,e^N}\xi_{e^N}U(c_{t,e^N}, l_{t,e^N})\right], \tag{22}$$

$$B_t + F(K_{t-1}, L_t, s_t) \geq G_t + (1+r_t)B_{t-1} + r_t K_{t-1} + w_t L_t, \tag{23}$$

for all $e^N \in \mathcal{E}^N$:

$$a_{t,e^N} + c_{t,e^N} \leq w_t\theta_{e_t^N} l_{t,e^N} 1_{e_0^N\neq 0} + \delta 1_{e_0^N=0} + (1+r_t)\tilde{a}_{t,e^N}, \tag{24}$$

$$\xi_{e^N}U_c\left(c_{t,e^N}, l_{t,e^N}\right) - \nu_{t,e^N} = \beta\mathbb{E}_t\left[\sum_{\tilde{e}^N\in\mathcal{E}^N}\Pi_{t+1,e^N,\tilde{e}^N}\xi_{\tilde{e}^N}U_c\left(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N}\right)(1+r_{t+1})\right], \tag{25}$$

$$l_{t,e^N} \geq \left(w_t\theta_{e_t^N}\right)^{\varphi} + \delta 1_{e_0^N=0}, \tag{26}$$

$$\nu_{t,e^N}(a_{t,e^N} + \overline{a}) = 0, \tag{27}$$

$$A_t = \sum_{e^N\in\mathcal{E}^N} S_{t,e^N}a_{t,e^N},\ L_t = \sum_{e^N\in\mathcal{E}^N} S_{t,e^N}\theta_{e_t^N}l_{t,e^N},\ K_t = A_t - B_t, \tag{28}$$

$$c_{t,e^N}, l_{t,e^N}, (a_{t,e^N} + \overline{a}) \geq 0, \tag{29}$$

with the law of motion (6) of $(S_{t,e^N})_{t\geq 0,e^N\in\mathcal{E}^N}$, and the definition (7) of $(\tilde{a}_{t,e^N})_{t\geq 0,e^N\in\mathcal{E}^N}$. Constraints (23)–(29) should be understood to apply to all $s^t \in \mathcal{S}^t$ and all $e^N \in \mathcal{E}^N$.[17]

Maximization devices in the Ramsey problem are both individual quantities – consumption level, labor effort, and asset holdings – and fiscal instruments: public debt, and post-tax interest and wage rates. Equation (23) is the government budget constraint, while the individual budget constraint is given in equation (24). The individual Euler equations for consumption and labor are provided in equations (25) and (26), respectively. The complementary slackness condition is stated in equation (27). Equation (28) gathers the aggregation for individual wealth and the labor supply, as well as financial market clearing. Finally, positivity and borrowing constraints appear in equation (29).

A reformulation of the Ramsey problem. We simplify the formulation of the Ramsey problem exposed in equations (22)–(29). We first denote by $\beta^t m^t(s^t)S_{t,e^N}\lambda_{t,e^N}$ the (discounted) Lagrange multiplier of the Euler equation of agent e^N in state s^t. We also define for all $e^N \in \mathcal{E}^N$:

$$\Lambda_{t,e^N} = \frac{\sum_{\hat{e}^N\in\mathcal{E}^N} S_{t-1,\hat{e}^N}\lambda_{t-1,\hat{e}^N}\Pi_{t,\hat{e}^N,e^N}}{S_{t,e^N}}, \tag{30}$$

[17] Again, $\mathbb{E}_t[\cdot]$ is the conditional expectation at date t with respect to aggregate shocks.

which, for agents of island e^N, can be interpreted as the average of their previous period Lagrange multipliers for the Euler equation. Finally, we note that $\lambda_{t,e^N} = 0$ if $a_{t,e^N} = -\overline{a}$: λ_{t,e^N} is zero when the credit constraint is binding. The product $\lambda_{t,e^N}\nu_{t,e^N}$ (for any t and any e^N) is thus always equal to 0. The following lemma summarizes our simplification of the Ramsey problem.

Lemma 1 (Simplified Ramsey problem) *The Ramsey problem in equations (22)–(29) can be simplified into:*

$$\max_{\left(r_{t+1},w_t,B_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N\in\mathcal{E}^N} S_{t,e^N} \Big(\xi_{e^N} U(c_{t,e^N}, l_{t,e^N}) \tag{31}$$

$$+ \xi_{e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N}\Big)\Big),$$

$$s.t.\ \lambda_{t,e^N} = 0 \ \textit{if}\ a_{t,e^N} = -\overline{a}, \tag{32}$$

and subject to equations (6), (7), (23)–(26), (28)–(29), and (30).

The proof is provided in Section C of the Appendix. The simplification of the Ramsey problem, which eases the computation of the maximization problem, is based on a re-writing of the Lagrangian to introduce Lagrange multipliers into the objective, as in Marcet and Marimon (2011). It could also provide a recursive formulation of the Ramsey problem that we do not need, as the sequential representation allows us to derive first-order conditions, expressed in a way that aids interpretation.

4.2 A benchmark: The complete market case

To understand how market incompleteness and heterogeneity affect the optimal tax system, we introduce the CM benchmark, extensively studied since Chamley (1986) and Judd (1985a). In the environment examined above, this corresponds to the case where all agents are on the same island. For ease of exposition, variables for the CM case will be denoted using an upper-script (0). For instance, $\mu^{(0)}$ denotes the Lagrange multiplier of the governmental budget constraint in the CM economy.

In the CM economy, complete insurance markets imply that the marginal utility of consumption is the same for all agents, no matter their history, which implies that we have $\xi_{e^N} U_c\left(c_{t,e^N}, l_{t,e^N}\right) = \xi_{f^N} U_c\left(c_{t,f^N}, l_{t,f^N}\right)$ for $e^N, f^N \in \mathcal{E}^N$. Furthermore, with the GHH utility function, we can easily find that $l_{t,e^N} = (\chi w_t \theta_{e^N})^{\varphi}$. These two equalities can be used to aggregate heterogeneity and to solve a standard representative agent problem.

Some of the results we present in both CM and IM economies depend on the convergence of the economy toward an interior steady state (where aggregate variables are constant). As shown by Straub and Werning (2014), this may not always be the case, because Lagrange multipliers may not converge toward finite or non-zero values. Recent contributions such as Chari, Nicolini, and Teles (2016) show that the behavior of

Lagrange multipliers depends on the set of instruments available to the planner. In addition, Chen, Chien, and Yang (2017) show that the steady state optimal tax is interior in a related model with incomplete-insurance markets. For the calibration provided in Section 5, we check that the interior steady-state is stable for both CM and IM economies. As a consequence, and to simplify the exposition, we compare steady-state outcomes of the CM and IM economies, assuming that they converge toward an interior steady-state.

The optimal fiscal system at the interior steady-state is easy to understand in this environment, following the analysis of Chari, Christiano, and Kehoe (1994) and Aiyagari, Marcet, Sargent, and Seppälä (2002) among others. In the CM allocation, households are borrowing at the maximum borrowing limit, and their debt is held by the government, which thus holds the highest possible quantity of assets (and thus a negative public debt). The government finances public spending out of interest payments made by firms and households. The key driver of this outcome is that the interest payments made by firms and households are not distorting and are thus an efficient way for the government to raise resources. The labor tax helps finance additional needs, when public spending is too high to be exclusively financed out of interest payments. The capital tax is zero at the steady-state, which is the traditional Chamley-Judd result. This environment is well known (see Chari, Christiano, and Kehoe 1994 or Sargent and Ljungqvist 2014 Chap. 15 for a textbook treatment), and we solve it in Appendix F. Finally, setting a non-zero credit constraint in the CM case is a requirement for having a non-zero tax base at the steady-state, which appears to be important for the dynamics investigated below.

4.3 Results for the incomplete-market economy

We formally derive the first-order conditions for the government in the IM economy in Section D of the Appendix. These equations – which are necessary conditions for the optimality of the Ramsey allocation – can be simplified by introducing a new intuitive concept, which we call the *social valuation of liquidity for agents* e^N and denote by ψ_{t,e^N}. It is formally defined as:

$$\psi_{t,e^N} \equiv \xi_{e^N} U_c(c_{t,e^N}, l_{t,e^N}) - \xi_{e^N} U_{cc}(c_{t,e^N}, l_{t,e^N}) \left(\lambda_{t,e^N} - (1+r_t)\Lambda_{t,e^N}\right). \tag{33}$$

The valuation ψ_{t,e^N} differs from the marginal utility of consumption $\xi_{e^N} U_c(c_{t,e^N}, l_{t,e^N})$ (which can be seen as the private valuation of liquidity for agents e^N) since ψ_{t,e^N} takes into consideration the saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$ and thus makes her Euler equation more "binding". This more "binding" constraint reduces the utility by the algebraic quantity $\xi_{e^N} U_{cc}(c_{t,e^N}, l_{t,e^N})\lambda_{t,e^N}$, where λ_{t,e^N} is the Lagrange multiplier of the agent's Euler equation at date t. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in Λ_{t,e^N}.

A similar expression can be derived in the CM economy. The social valuation of liquidity for the representative agent can be written as:

$$\psi_t^{(0)} = \bar{\xi}_t U_c(c_t^{(0)}, l_t^{(0)}) - \bar{\xi}_t U_{cc}(c_t^{(0)}, l_t^{(0)}) \left(\lambda_t^{(0)} - (1+r_t^{(0)})\lambda_{t-1}^{(0)}\right), \tag{34}$$

where $\bar{\xi}_t$ is a new parameter coming from heterogeneous preferences and whose expression is made explicit in Appendix F. Note that the expressions in the IM and CM economies, (33) and (34), respectively, are analogous. We now present and discuss the first-order conditions of the planner using these concepts for the three instruments: labor tax, capital tax, and public debt.

Labor tax. In the CM economy, the optimal labor tax is characterized by:

$$\varphi \frac{\tau_t^{L,(0)}}{1-\tau_t^{L,(0)}} = 1 - \frac{\psi_t^{(0)}}{\mu_t^{(0)}}. \tag{35}$$

The left-hand side is a measure of the marginal cost of raising resources with the labor tax, taking into account distortions, which are an increasing function of labor elasticity φ. The right-hand side is a measure of the marginal gain. If the government and households value liquidity identically, i.e. if $\psi_t^{(0)} = \mu_t^{(0)}$, then the right-hand side is null, and so is the labor tax $\tau^{L,(0)}$. There is no use for a distorting tool. Conversely, when the government has a greater liquidity need than that of households: $\mu_t^{(0)} > \psi_t^{(0)}$, the labor tax becomes positive. From Chari, Christiano, and Kehoe (1994), and confirmed in our simulations below, it is known that both valuations $\psi_t^{(0)}$and $\mu_t^{(0)}$ move closely together along the business cycles in the CM economy. As a consequence, the labor tax remains very stable after either a technology or a public spending shock.

In the IM economy, the tax is defined by a similar equation. Before turning to the formal equation, we define the efficient labor share of households with history e^N as $\omega_{t,e^N}^L \equiv \frac{S_{t,e^N} l_{t,e^N} \theta_{e_t^N}}{L_t}$, which represents the share of workers with history e^N in the labor-tax base at date t. Note that $\sum_{e^N\in\mathcal{E}^N} \omega_{t,e^N}^H = 1$. The restriction on the labor tax is:

$$\varphi \frac{\tau_t^L}{1-\tau_t^L} = 1 - \frac{\sum_{e^N\in\mathcal{E}^N} \omega_{t,e^N}^L \psi_{t,e^N}}{\mu_t}. \tag{36}$$

The two expressions (35) and (36) for the CM and IM economies, respectively, are very similar. The distortions induced by the labor tax are equalized to the equilibrium gain of transferring resources from all households to the government. The household valuation of liquidity is now an average across households, which can be written as $\sum_{e^N\in\mathcal{E}^N} \omega_{t,e^N}^L \psi_{t,e^N} = \sum_{e^N\in\mathcal{E}^N} \psi_{t,e^N} + cov_{e^N}(\omega_{t,e^N}^L, \psi_{t,e^N})$. The covariance term (across histories) highlights an additional net cost of using the labor tax, stemming from its redistributive effect. If the covariance is negative (as in the case in the quantitative investigation below), households with a high labor income have a low liquidity need. In this case, the labor tax tends to be progressive and has a small redistributive cost. As a result, when the covariance becomes increasingly negative, the labor tax will rise, all other things being constant. The reverse holds when the covariance is positive.

It is noteworthy that the labor tax is not a sufficient statistic for the distortions in the economy. CM and IM economies can in fact face similar labor taxes that give rise to very different distortions. The two ratios $\frac{\psi_t^{(0)}}{\mu_t^{(0)}}$ and $\frac{\sum_{e^N \in \mathcal{E}^N} \omega_{t,e^N}^L \psi_{t,e^N}}{\mu_t}$ can be similar with $\sum_{e^N \in \mathcal{E}^N} \omega_{t,e^N}^L \psi_{t,e^N}$ much higher than $\psi_t^{(0)}$, if it is also the case for μ_t and $\mu_t^{(0)}$.

Capital tax. In the CM economy, the first-order condition for the capital tax can be expressed as:

$$\frac{\bar{\xi}_t U_c(c_t^{(0)}, l_t^{(0)}) \lambda_{t-1}^{(0)}}{\mu_t{}^{(0)} A_{t-1}^{(0)}} = 1 - \frac{\psi_t^{(0)}}{\mu_t{}^{(0)}}. \tag{37}$$

The first order condition (37) for the capital tax closely parallels that (35) for the labor tax. This equation states that the (intertemporal) cost of of transferring one unit of resources to the government (left-hand side) is equal to the marginal benefit. The intertemporal distortion generated by the capital tax is a decreasing function of the capital stock, as one additional unit of resources is generated by a smaller marginal increase in the capital tax when the capital tax base, A_{t-1}, is higher. The distortion is also an increasing function of the Lagrange multiplier $\lambda_{t-1}^{(0)}$ as it affects capital accumulation from periods $t-1$ to t.

Before turning to the first-order condition in the IM economy, we first define, as we did in the labor tax case, $\omega_{t,e^N}^K \equiv \frac{S_{t,e^N} \tilde{a}_{t,e^N}}{A_{t-1}}$. This is the share of households with history e^N in the period-t tax base. The first-order condition in the IM economy can now be expressed as

$$\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \frac{U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N}}{\mu_t A_{t-1}} = 1 - \frac{\sum_{e^N \in \mathcal{E}^N} \omega_{t-1}^K \psi_{t,e^N}}{\mu_t}. \tag{38}$$

Once again, first-order conditions (37) and (38) have a similar expression in the CM and IM cases. In the CM case, the cost of levying resources depends on the term $\sum_{e^N \in \mathcal{E}^N} \omega_{t-1}^K \psi_{t,e^N} = \sum_{e^N \in \mathcal{E}^N} \psi_{t,e^N} + cov_{e^N}(\omega_{t,e^N}^K, \psi_{t,e^N})$, where the covariance term again captures the redistributive effect of the capital tax. The more negative the covariance term, the less costly it becomes to levy resources with capital tax. The covariance term is negative (as it is in our quantitative investigation below) when wealthier households have lower liquidity needs.

Public debt. The first-order condition for public debt in the IM economy pinning down the dynamics of μ_t is:

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}\right)\right]. \tag{39}$$

Equation (39) sets equal the marginal benefit of one additional unit of debt at date t to the marginal extra cost at date $t+1$, using the before-tax return $\tilde{r}_t$ to value the next period. This expression is exactly the same as in the CM economy, and so we omit it. Public debt is a residual in the CM economy that can be obtained from the government's budget constraint. The rationale for public debt dynamics is that they can smooth tax distortions, but the precise dynamics are hard to characterize theoretically, as they depend on the volatility of taxes and therefore on underlying shocks (Chari, Christiano, and Kehoe 1994). The same reasoning prevails in the IM economy, as will be shown in the quantitative analysis below.

Steady-state capital tax. The first-order condition of the planner program enables us to derive additional results regarding the steady-state fiscal policy. The main results are gathered in the following proposition.

Proposition 3 (Steady-state) *In the interior steady-state of the Ramsey equilibrium:*

1. *the marginal productivity of capital is determined by the discount factor β:*

$$1 + F_K(K, L) = \frac{1}{\beta}, \tag{40}$$

2. *the capital tax is non-negative; it is positive if, and only if, credit-constraints bind for some agents, or more formally:*

$$\tau^K = \frac{\sum_{e^N \in \mathcal{C}} S_{e^N} \nu_{e^N}}{(1-\beta) \sum_{e^N \in \mathcal{E}^N} S_{e^N} U_c\left(c_{e^N}, l_{e^N}\right)}, \tag{41}$$

where we recall that $\mathcal{C}$ is the set of islands where credit constraints bind at the steady-state, and ν_{e^N} is the Lagrange multiplier of credit constraint for island e^N.

The first item in equation (40) of Proposition 3 is a direct implication of the government Euler equation (39). As a consequence, the marginal productivity of capital is determined solely by the discount factor β, as originally explained by Aiyagari (1995). This important restriction is the so-called "modified golden rule". The second item in Proposition 3 is new in the literature in this form.[18] The capital tax is always non-negative and its value is determined by the Lagrange multipliers on the households' credit constraints. The steady-state capital tax will be positive if, and only if, some agents are credit-constrained. In this case, some agents do not internalize the effect of

[18]This expression does not depend on truncation, and we checked that the same expression can be obtained in the steady state in the case where $N = \infty$.

the interest rate on their saving decision. They thus save to self-insure, generating an overall oversupply of liquidity, as identified by Woodford (1990).[19] In other words, it is not market-incompleteness but binding credit constraints that justify positive capital taxation at the interior steady-state.

In the previous analysis, we assumed that the credit constraint was tighter than the natural borrowing limit, which is the case in most quantitative works. The motivation for this assumption is the previous capital tax result, which raises a difficulty in the case where the credit constraint is below the natural borrowing limit. In this case, no household is credit-constrained, the capital tax is 0 (Item 2 of the Proposition),[20] and households thus face a post-tax interest rate of $1 + r = \frac{1}{\beta}$ (item 1). In this case, an interior steady-state equilibrium may not exist when N becomes large enough, because household savings tend toward infinity, while the capital stock is finite. This was already discussed in Bewley (1983) in a monetary environment, and in Aiyagari (1995) with capital accumulation.[21]

5 Quantitative properties of the optimal tax system

We now provide a quantitative investigation of the optimal tax system, after both a technology and a public spending shock, to further understand the effect of market incompleteness.

5.1 Parameter calibration

We first provide a calibration for standard parameters, before explaining the choice of the truncation length N and of taste shocks $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$.

First, the utility function is $u\left(c - \chi^{-1}\frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) = \log\left(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$, with a Frisch elasticity of labor supply set to $\varphi = 0.5$, which is consistent with empirical estimates (Chetty, Guren, Manoli, and Weber 2011). The period is a year, and the discount factor is $\beta = 0.96$.

[19] Davila et al. (2012) use perturbation methods to show that capital stock can be either too high or too low in the steady state. We characterize the capital tax as a tool used to affect capital accumulation. The proposition states that this tax is not used to subsidize capital accumulation. Note that, in our analysis, this result depends on the commitment assumption. See Benhabib and Rustichini (1997) for the case without commitment and with complete markets. Finally, Piketty and Saez (2013) also argue in their Appendix A.3 that capital tax may be positive if some households do not leave a bequest, which is the equivalent of credit constraints being binding in our environment.

[20] The denominator in equation (41) is bounded away from zero because the economy is finite as the marginal productivity of capital is bounded away from 0 (from item 1 of the Proposition), i.e., not all marginal utilities can be simultaneously zero.

[21] Note that Aiyagari (1995) proves that *if* an interior steady-state equilibrium exists, then capital tax is positive in a Bewley model with capital accumulation. The difficulty is that such an equilibrium may not exist. The structure of the equilibrium with optimal taxes, capital accumulation, and credit constraints below the natural borrowing limit is left for future research.

The production function is Cobb-Douglas with a constant capital depreciation ς: $F(K, L, z) = \Psi(z)K^\alpha L^{1-\alpha} - \varsigma K$, where $\Psi(z) = \exp(z)$ is the technology level. As is standard, the capital share is set to $\alpha = 0.33$, and the depreciation rate to $\varsigma = 10\%$. The technology shock is assumed to follow an AR(1) process: $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $(\varepsilon_t^z)_{t\geq 0}$ is a white-noise process with a distribution $\mathcal{N}(0, \sigma_z^2)$. We set the annual persistence of the technology shock to $\rho_z = 0.81$, which corresponds to a quarterly persistence of 0.95. The standard deviation is set to $\sigma_z = 1\%$. Public spending is assumed to be of the form $G_t = G\exp(g_t)$, where g_t follows an AR(1) process: $g_t = \rho_g g_{t-1} + \varepsilon_t^g$, where $(\varepsilon_t^g)_{t\geq 0}$ is a white-noise process with a distribution $\mathcal{N}(0, \sigma_g^2)$. We set the annual persistence of the technology shock to $\rho_g = 0.89$ and the standard deviation to $\sigma_g = 7\%$, following Chari, Christiano, and Kehoe (1994) and Farhi (2010).

We calibrate the idiosyncratic risk to the unemployment risk, as in Imrohoroğlu (1992) or Krusell and Smith (1998) among others. Households can be either employed, in which case they choose their labor supply, or unemployed, in which case they obtain δ consumption units from home production. For employment risk, we derive transition probabilities using a calibration based on the strategy of Shimer (2003). The annual transition matrix is $M = \begin{bmatrix} 0.057 & 0.943 \\ 0.056 & 0.944 \end{bmatrix}$.[22] The home production parameter δ is set such that home production amounts to 50% of market income. Finally, steady-state public spending is set to $G = 0.38$, which implies a steady-state public spending-to-GDP ratio roughly equal to 33%, which is US average public spending in early 2000 net of unemployment benefits, to be consistent with the model.[23]

The credit limit is set to $\bar{a} = 0.01$. In incomplete market models, this credit limit is often set to 0 for computational speed. However, as was explained above, the case $\bar{a} = 0$ happens to be a very special case when deriving optimal fiscal policy under complete markets, as the steady-state capital tax base is 0. This small deviation from the standard case is sufficient to properly characterize the properties of fiscal policy under complete markets, so as to be able to compare them with our results under incomplete markets.

Table 1 summarizes our calibration for standard parameters.

β	ϕ	α	ς	$\delta/(wl)$	G	ρ_z	σ_z	ρ_g	σ_g	$\bar{a}$
0.96	0.5	0.33	10%	50%	0.38	0.81	0.01	0.89	0.07	0.01

Table 1: Parameter values

[22] More precisely, we consider the quarterly matrix derived by Challe and Ragot (2016) compounded four times to obtain annual transitions. Roughly, the quarterly job finding rate is 80% and the quarterly job separation rate is 5%.

[23] An alternative strategy would be to target government consumption, which is lower than public spending. This may under-estimate the distortions of the tax system. For our exercise, the financing of total spending seems to be a more relevant target.

Choosing N and $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$. The remaining coefficients are the truncation parameter N, which characterizes the magnitude of insurance – through pooling – for the idiosyncratic risk, and the preference parameters $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$. As we consider an unemployment risk with two states, we have 2^N different individual histories. We must also calibrate the 2^N history-dependent preference parameters $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$. Although the model can be easily simulated for the case $\xi_{e^N} = 1$ for all e^N, we nevertheless use the model's flexibility to calibrate these parameters to match a specific distribution. The interest of this strategy is twofold. First, it provides a solution that makes the model more quantitatively relevant, which is of independent interest. Second, this strategy also fastens the convergence of the optimal tax system, especially for small values of the parameter N, which helps speed up the convergence.

How do we calibrate $(\xi_{e^N})_{e^N}$? We begin by simulating a Bewley economy (i.e., without aggregate shock $\Psi = 1$ and $G_t = G$) for the same period utility function but with $\xi_{e^N} = 1$ for all $e^N \in \mathcal{E}^N$ and for exogenous prices, to generate a benchmark distribution of wealth. For our simulation, we consider a post-tax annual interest rate of $r = 4\%$ and a post-tax wage rate of $w = 0.684$. This last value is consistent with the Cobb-Douglas production function calibrated with the values of Table 1, together with a realistic tax system.[24] Then, for a given N, we simulate the island economy with the same exogenous prices and determine the $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$ – with the normalization $\sum_{e^N \in \mathcal{E}^N} S_{e^N} \xi_{e^N} = 1$ – such that the distribution of wealth in the island economy is close to that of the Bewley economy (see Section G of the Appendix for details of the computational method).[25]

For any given N, we can therefore determine the $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$ and deduce the wealth equilibrium distribution (with endogenous prices), as well as the amount of risk sharing. We proxy risk sharing by the standard deviation across agents of the "pooling" transfers $\Gamma^*(e^{N+1})$, normalized by the total income $Inc_{e^{N+1}}$ of agents with history e^{N+1} ($Inc_{e^{N+1}} = w\theta_{e_0^{N+1}} l_{e^{N+1}} + \delta 1_{e_0^{N+1}=0} + (1+r)a_{e^{N+1}}$), which will be denoted by sd_Γ. A value equal to 5% means that the standard deviation of the value of pooling transfers relative to agents' own *individual* income is 5%, which is low.

In Table 2, we report for different values of N the wealth distribution (in deciles, noted from D1 to D10), the standard deviation sd_Γ, and the standard deviation of the taste shocks $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$, which we denote by sd_ξ. For each wealth decile, we report the cumulative share of the total wealth held by agents in the deciles up to and including the given decile (so as to provide a simplified Lorenz curve). For instance, column D6 corresponds to the share of total wealth held by agents belonging to the first 6 deciles.

[24] More precisely, we consider a capital tax $\tau^K = 36\%$ and a labor tax $\tau^L = 28\%$ (taken from Trabandt and Uhlig 2011) and we define w by $w = \left(1 - \tau^L\right) \tilde{w} = \left(1 - \tau^L\right) \left(\frac{\alpha}{\frac{r}{1-\tau^K} + \mu} \right)^{\frac{\alpha}{1-\alpha}}$.

[25] We use a general algorithm that derives results for any N. Admittedly, for a given truncation N, distributions could be more closely matched by using more elaborated methods such as simulated methods of moments. We leave these improvements for future work.

N	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	sd_Γ (%)	sd_ξ (%)
4	2.13	6.44	17.74	29.49	41.24	52.99	64.75	76.50	88.25	100	8.40	0.69
5	2.17	6.65	15.48	27.55	39.63	51.70	63.78	75.85	87.92	100	7.42	0.70
6	2.18	6.76	13.39	25.76	38.13	50.50	62.88	75.25	87.63	100	6.65	0.72
7	2.24	6.90	13.33	24.20	36.84	49.47	62.10	74.73	87.37	100	5.95	0.72
7 ($\xi = 1$)	-0.48	2.98	8.76	19.79	33.16	46.53	59.90	73.26	86.63	100	1.48	0
∞	1.49	6.42	13.20	21.53	31.30	42.50	55.04	68.87	83.87	100	-	-

Table 2: Simulation results. D1 to D10 represent deciles in the cumulative wealth distribution. sd_Γ is the standard deviation $sd(\Gamma^*(e^{N+1})/Inc_{e^{N+1}})$, while sd_ξ is the standard deviation of $(\xi_{e^N})_{e^N}$ (see the text for further details).

We report all of these quantities for various values of N, starting with $N = 4$ for the sake of conciseness. We stop the simulations at $N = 7$, because the optimal tax system has roughly converged (see Table 3 below). Finally, in the last row we report the distribution generated by the Bewley model, labeled ∞.

From Table 2, we observe that as N rises, the wealth distribution gets closer to the Bewley distribution and that the standard deviation of the pooling transfer falls. This confirms that as N rises, the idiosyncratic risk sharing provided by asset pooling falls. For $N = 7$, the standard deviation of pooling transfers is relatively low, and below 6%. To provide a comparison, we also report the simulated distribution for $N = 7$ and for $\xi_{e^N} = 1$ (labeled as row "7($\xi = 1$)"). In this case, poor agents hold too little wealth. Finally, the standard deviation of the taste shocks $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$ remains low, and below 1% for all simulations.

Numerical methods. To solve the model, we first solve for the steady state. This is not difficult, as the above equations define an almost linear system. We provide the Algorithm in Section H of the Appendix. Second, we write a code that writes the set of dynamic equations in Dynare for an arbitrary N. This allows us to use the Dynare solver to double-check our steady-state computations and to simulate the model. We check that the standard stability and credit-constraint conditions are fulfilled and that all variables converge back to their steady state value after the shocks. Simulating the model takes a couple of seconds once the steady-state is found.

5.2 Steady-state tax system

We simulate the island economy – with properly calibrated taste shocks for each N – to determine the steady state optimal fiscal system, consisting of capital τ^K and labor taxes

τ^L, and the optimal public debt. We report the results in the first part of Table 3 for different values of N, including $N = 0$, which corresponds to the complete market case discussed in Section 4.2. For the debt, we report the debt-to-GDP ratio B/Y. We also report the standard deviation of the normalized pooling transfers sd_Γ, which differ from those in Table 2, because the tax system and prices are different.

N	τ^K (%)	τ^L (%)	B/Y	sd_Γ (%)
0	0.0	34.5	−2.35	–
4	7.4	34.9	−2.29	1.6
5	7.5	34.9	−2.29	1.3
6	7.6	34.9	−2.29	1.0
7	7.6	34.9	−2.29	0.7
Sensitivity analysis				
7 ($\xi = 1$)	7.8	34.8	−229	1.3
7 ($\delta/wl = 0.55$)	5.1	34.9	−230	0.7
7 ($\phi = 0.6$)	7.1	34.8	−230	0.5

Table 3: Steady-state optimal fiscal system

In the complete-market case ($N = 0$), public debt is negative. As already explained in Section 4.2, the government holds all the capital stock and finances its residual financial needs with labor taxes, which amount to $\tau^L = 34.5\%$. Capital taxes are zero, as is standard in complete-market environments.[26] The fiscal system converges rapidly with N and the standard deviation of the pooling transfer is low, and smaller than 1% for $N = 7$. We now discuss the results for the case $N = 7$. First, the labor tax amounts to 34.9%, which is slightly higher than with complete markets (34.5%). The government does not significantly rely on the labor tax for redistribution, because distortions are high in the incomplete-market economy. To see this, we need to use equations (35) and (36) defining the labor tax in complete and incomplete economies. Labor tax rates are very similar in both types of economy: the ratio $\frac{\sum_{e^N \in \mathcal{E}^N} \omega^L_{e^N} \psi_{e^N}}{\mu^{(N)}} = 0.73$ in (36) for IM is close to the $\frac{\psi^{(0)}}{\mu^{(0)}} = 0.74$ in (35) for CM. However, the close tax rates mask significant differences between IM and CM. For example, both the need to levy resources and the distortions generated by the labor tax are higher in the IM economy than in CM economy. We have $\sum_{e^N \in \mathcal{E}^N} \omega^L_{e^N} \psi_{e^N}(2.85) > \psi^{(0)}(2.79)$ for household liquidity needs and $\mu(3.92) > \mu^{(0)}(3.79)$ for governmental liquidity needs: both households and governments have higher liquidity needs. The redistribution motive tends to increase the labor tax, because the covariance

[26]We checked whether the steady-state was an interior solution by studying the government's optimal policy. It was indeed found to converge back to the steady state after both a technological and a public spending shock.

is negative $corr(\omega^L_{e^N}, \psi_{e^N})(-0.40) < 0$, but the high level of household liquidity needs $\sum_{e^N \in \mathcal{E}^N} \omega^L_{e^N} \psi_{e^N}(2.85)$ makes tax labor costly to use and limits its magnitude.

Second, the capital tax is zero as expected in the CM economy, and it is small but positive (7.6%) in the IM economy. In the IM economy, a positive though small number of agents (roughly 5%) face a binding credit constraint. Households save too much in this case, because credit-constrained households do not internalize the interest rate effect into their borrowing decision.

Finally, the public debt-to-GDP ratio is 6 percentage points higher in IM than in CM. Following the analysis of Woodford (1990), this difference stems from the fact that public debt is a useful device that enables private agents to self-insure in the absence of complete insurance markets. The cost of higher (i.e. less negative) public debt is that more resources must be obtained from distorting tools, as in Aiyagari and McGrattan (1998).[27]

Sensitivity analysis. The second part of Table 3 provides a sensitivity analysis for three sets of parameters for $N = 7$. First, we provide the optimal fiscal system for all $\xi_{e^N} = 1$ (everything else is held constant) in line 7. The capital tax is a little higher and the labor tax a little lower, whereas public debt does not vary. Thus, the fiscal system does not significantly depend on the matching of the wealth distribution. Second, we increase home production from 50% to 55% of labor income in line 8. As there is less need to self-insure, capital tax falls, as does public debt (there is less liquidity to self-insure). The labor tax remains constant, because there is little net effect on liquidity needs. Third, we increase the Frisch elasticity to 0.6, instead of 0.5. To make the economy comparable, we adjusted the labor supply parameter ($\chi = 1.07$) to ensure the same average steady-state labor supply. The steady-state labor tax is lower (34.8%) than in the benchmark (34.9%). For our preference, this change reduces the difference in marginal utility between employed and unemployed agents. As a consequence, there is a lower insurance need and both public debt and capital tax fall. Finally, the dynamics of the fiscal system that we report below for the benchmark calibration do not significantly vary in the sensitivity analysis we consider.

5.3 Negative public spending shock

We now simulate the optimal change in the fiscal system after a negative public spending shock (to obtain an expansion in all our simulations). We consider an unexpected fall in the ratio of public spending-to-GDP of 1 percentage point and then let public spending

[27]Note that Aiyagari and McGrattan (1998) determine the optimal level of public debt by numerically maximizing steady-state welfare, whereas we solve for the steady-state of the Ramsey problem, which is a different exercise. For this reason, it is not possible to compare the optimal level of debt for these different exercises.

converge back to its equilibrium value.

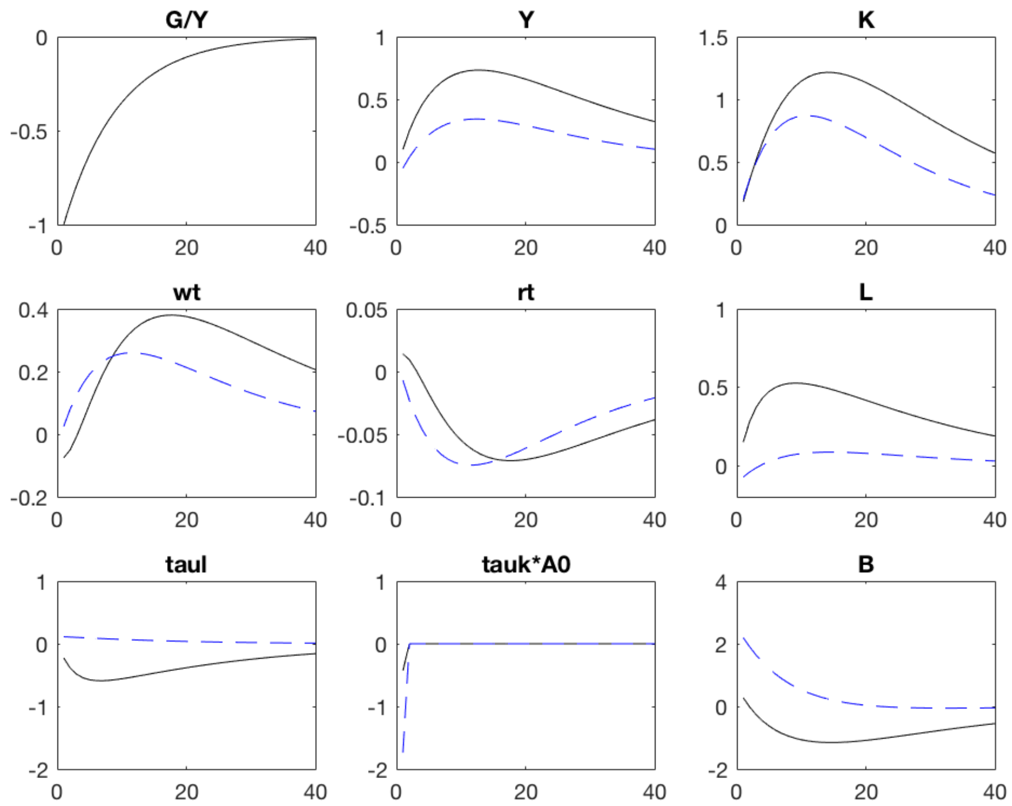

Figure 1: Aggregate IRFs after a 1 pp decrease in the ratio of public-spending-to-GDP (G/Y). See the text for a description of the variables. Solid line: IM economy with $N = 7$; dashed blue line: CM economy.

We compare the responses in IM and CM economies (for $N = 7$) and report the results in Figure 1, where we focus on key variables in order to understand the main mechanisms. The first line of panels plots the shock (G/Y), the GDP (Y), and the capital stock (K). The second line represents the before-tax wage rate (wt), the before tax real interest rate (rt), and total labor (L). The third line plots the government's fiscal instruments: the labor tax (taul), a transformation of the capital tax (tauk*A0, which we discuss below), and public debt (B). All variables are given in proportional deviation from steady-state values, except for taxes and the interest rate, where we report level deviations. For each variable, the dashed blue line corresponds to CM, and the solid black line to IM.

The outcome of the CM economy is now well understood (see Chari, Christiano, and Kehoe 1994, among others). The reduction in public spending is a positive wealth shock. The government transfers wealth back to households with an initial large change in the capital tax for one period, as this is not distorting. The labor tax is almost unaffected during these dynamics. For the sake of interpretation, the capital tax is represented in Figure 1 as the tax rate multiplied by the steady-state total household saving: $\tau_t^K \times A_{ss}^{(0)}$

where $A_{ss}^{(0)} = -0.01$, to aid interpretation. A fall in this product means that resources are transferred to households. Indeed, as households hold a negative amount of assets, an increase in the capital tax τ^K (and thus a reduction in the product) actually *reduces* households' interest payment, enabling the government to transfer resources to households.[28]

The main difference in the IM economy is that the government reduces the labor tax to transfer resources to households. This contributes to an increased labor supply (L) and thus further drives up the GDP (Y) compared to the CM economy. By examining the mechanisms pinning down the labor tax in equations (35) and (36), we observe that the fall in the labor tax stems from a bigger fall in μ_t compared to $\sum_{e^N \in \mathcal{E}^N} \omega_{e^N,t}^L \psi_{t,e^N}$, whereas the fall in $\mu^{(0)}$ is the same order of magnitude as the fall $\psi_t^{(0)}$ in CM. In other words, in the IM economy, the government's liquidity need is higher in the steady state compared to the CM economy, but falls more after the shock compared to the fall in agents' needs. This incentivizes the government to reduce labor tax.

Another feature of the IM economy is that public debt is countercyclical, while it is procyclical in the CM economy. Public debt is used in the CM economy to frontload the transfer to households, through the capital tax, and to smooth out tax distortions. In the IM economy, there is no such frontloading, as the labor tax falls during the dynamics.

Concerning the capital tax, in the IM economy, steady-state savings are positive ($A_{ss} = 0.05$). The product $\tau_t^K \times A_{ss}$ is lower because capital tax falls to transfer resources to households. In addition, capital tax generates some redistributive effects, which makes capital tax less attractive for transferring wealth to households in the IM economy. The correlation measuring the redistributive effects of capital tax in the IM economy, $corr(\omega_{e^N}^K, \psi_{e^N})$, is negative ($-0.41$ in the steady state) (see equation 38). In other words, transferring resources though the capital tax means transferring large resources to agents who have low liquidity needs. It is therefore regressive.

5.4 Positive TFP shock

We now study the impact on the fiscal system of a positive TFP shock. Figure 2 plots the same variables as in Figure 1, except in the first panel, which plots the TFP shock in both economies.

First, the mechanisms in the CM economy are similar to those following a drop in public spending, discussed above. The economy experiences a positive wealth effect and the government transfers resources using the capital tax, while the labor tax barely moves. Public debt is used to smooth out tax distortions.

The striking result is that the IM outcome is now very similar to the CM one. The

[28] This transformation avoids representing an increase in taxes as a reduction in interest payments. We can thus easily find the time path of the capital tax from the quantity $\tau_t^K \times A_{ss}^{(0)}$.

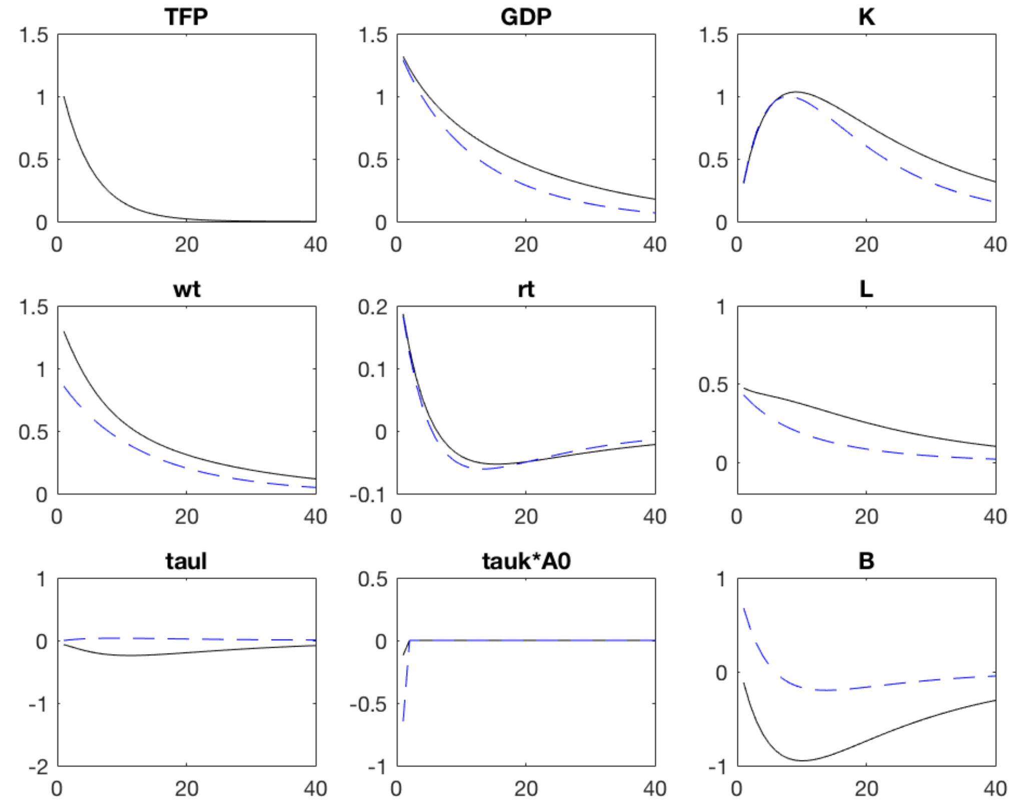

Figure 2: Aggregate IRFs after a 1 pp increase in TFP. See the text for a description of the variables. Solid line: IM economy with $N = 7$; dashed blue line: CM economy.

labor tax (taul) barely moves in the IM economy, such that the labor supply (L) is close to that of the CM economy. Examining the mechanisms reveals that the fall in the governmental budget Lagrange multiplier, μ, is higher in the IM than in the CM, but that this is offset by a fall of the same order of magnitude in $\sum_{e^N \in \mathcal{E}^N} \omega^L_{e^N} \psi_{e^N}$. Overall, the government's reduced liquidity need is accompanied by the similarly reduced liquidity needs of households, which allows the labor tax to remain largely unchanged. The only difference between the CM and IM economies is related to the response of public debt, which is countercyclical in the IM economy and procyclical in the CM economy.

Summarizing our results, we first observe that capital taxes are very volatile in the CM and IM economies for both shocks, as they represent a non-distorting way of transferring resources to households (at least on impact). For the CM case, this finding was already present in Chari and Kehoe (1999) and Farhi (2010). This happens to also be the case for incomplete insurance markets. Second, the main difference between the IM and CM economies stems from the labor tax, which is more volatile in the IM economy because of redistribution constraints. Third, the difference between the IM and CM economies is particularly significant after a public spending shock, but less so after a TFP shock.

Finally, public debt in the IM and CM economies exhibits different dynamics for the two shocks, as public debt can be used to frontload transfers to households in the CM economy, while this is less the case in the IM economy.

6 Concluding remarks

The market equilibrium in an incomplete insurance-market economy with aggregate shocks can be represented as the allocation of a family-head program. We use this representation to generate a finite-dimensional state-space equilibrium, in which the Ramsey outcome can be studied with aggregate shocks and various fiscal tools. We employ this framework to study optimal fiscal policy when positive transfers, distorting taxes on capital and labor, and public debt are available. The first interest of this framework is that it can be used to analytically derive various properties of the tax system, such as the steady-state level of capital and labor taxes. A second interest is its ability to simply simulate these economies. We apply this quantitative investigation to a standard economy, where the employment risk is uninsurable, and show that the dynamics of the fiscal system are different when agents are heterogeneous, compared to the representative agent case. We investigate the convergence of this economy as a function of N and show that a reasonably small value of N is sufficient to capture the main characteristics of the fully-fledged incomplete market economy.

The methodology presented in this paper could be applied to different settings with incomplete insurance-markets and aggregate shocks. In the general case, these environments are difficult to analyze and our simplified theoretical representation could help to derive new results. We thus consider our truncated representation as a complement to alternative approaches. To take a concrete example, we could first increase the number of instruments available to the government, such as non-linear tax schedules to model progressive or regressive fiscal systems, or explicit unemployment benefits, considering capital accumulation. Second, additional heterogeneity, such as the qualification structure or limited participation in financial markets to model wealthy hand-to-mouth households could be introduced to make more extensive use of empirical estimates of key parameters. Third, and just as importantly, it is also possible to include other distortions, such as nominal frictions or frictional labor markets, in order to derive optimal monetary policy in these richer environments.

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Appendix

A Proof of Proposition 1

Consider an agent endowed with the $N+1$-period history $e^{N+1} = (\hat{e}^N, e) \in \mathcal{E}^{N+1}$, which can also be written as $e^{N+1} = (e_N, e^N)$. In the former notation, e^{N+1} is seen as the history $\tilde{e}^N \in \mathcal{E}^N$ with the successor state $e \in \mathcal{E}$, while in the latter notation, e^{N+1} is seen as the state $e_N \in \mathcal{E}$ followed by history $e^N \in \mathcal{E}^N$. The solutions to the program (18)–(20) are the policy rules $c = g_c^{N+1}(a, e^{N+1})$, $a' = g_{a'}^{N+1}(a, e^{N+1})$, $l = g_l^{N+1}(a, e^{N+1})$ and the multiplier $\nu = g_\nu^{N+1}(a, e^{N+1})$ satisfying:

$$U_c(c,l) = \beta \mathbb{E}\left[\sum_{e' \in \mathcal{E}} M_{e,e'} U_c(c', l')(1+r)\right] + \nu, \tag{42}$$

$$l = (\chi w \theta_e)^{\varphi} 1_{e>0} + \delta 1_{e=0}, \tag{43}$$

$$\nu(a' + \bar{a}) = 0 \text{ and } \nu \geq 0. \tag{44}$$

We use a guess-and-verify strategy. The transfer is constructed such that all agents with the same N-period history have the same after-transfer wealth. The measure of agents with history e^N follows the same law of motion as (6) in the island economy and is equal to S_{e^N}. If agents with the same history $(\hat{e}^N, e), e \in \mathcal{E}$ have the same beginning-of-period wealth $a_{\hat{e}^N}$, the after-transfer wealth, denoted by $\hat{a}_{e^N}$, of agents with history $e^N \succeq \hat{e}^N$ is:

$$\hat{a}'_{e^N} = \sum_{\tilde{e}^N \in \mathcal{E}^N} \frac{S_{\tilde{e}^N}}{S_{e^N}} \Pi_{\tilde{e}^N, e^N} a'_{\tilde{e}^N}, \tag{45}$$

such that agents with the same history hold the same wealth. By construction, $\hat{a}_{e^N}$ follows dynamics similar to the "after-pooling" wealth $\tilde{a}_{t,e^N}$ in the island economy of equation (7). The transfer scheme denoted by $\left(\Gamma^*_{N+1}(e^{N+1})\right)_{e^{N+1} \in \mathcal{E}^{N+1}}$ that enables all agents with the same history to have the same wealth is:

$$\Gamma^*_{N+1}(e^{N+1}) = (1+r)\left(\hat{a}_{e^N} - a_{\hat{e}^N}\right), \tag{46}$$

where we use $e^{N+1} = (\hat{e}^N, e) = (e_N, e^N)$. The transfer $\Gamma^*_{N+1}(e^{N+1})$ defined in (46) replaces the beginning-of-period wealth $(1+r)a_{\hat{e}^N}$ with the *average* wealth $(1+r)\hat{a}_{e^N}$. Since there is a continuum with mass $S_{\tilde{e}^N}$ of agents with history $\tilde{e}^N$, in which each individual agent is atomistic, all agents take the transfer Γ^*_{N+1} as given.

Finally, it is easy to check that the transfer scheme is balanced in each period. Using the definition (45) of $\hat{a}_{e^N}$, we obtain for $e^N = (e^N_{N-1}, \ldots, e^N_1, e^N_0) \in \mathcal{E}^N$, $S_{e^N}\hat{a}_{e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} S_{\hat{e}^N} \Pi_{\hat{e}^N, e^N} a_{\hat{e}^N} = \sum_{\hat{e} \in \mathcal{E}} S_{(\hat{e}, e^N_{N-1}, \ldots, e^N_1)} M_{e^N_1, e^N_0} a_{(\hat{e}, e^N_{N-1}, \ldots, e^N_1)}$. Therefore, we deduce: $\sum_{\tilde{e} \in \mathcal{E}} S_{(\tilde{e}, e^N)} \Gamma^*_{N+1}(\tilde{e}, e^N) = (1+r)\left[\sum_{\tilde{e} \in \mathcal{E}} S_{(\tilde{e}, e^N)}\left(\hat{a}_{e^N} - a_{(\tilde{e}, e^N_{N-1}, \ldots, e^N_1)}\right)\right] = 0$, where the last equality comes from the definition of $\hat{a}_{e^N}$ in equation (45).

B Proof of Proposition

The proof comprises three steps. In the remainder, we use the following notation. For $N > k > 0$, $e^k = (e_{k-1}, \ldots, e_0) \in \mathcal{E}^k$, $e^{N,k} = (e_N, \ldots, e_k) \in \mathcal{E}^{N+1-k}$, and $(e^{N,k}, e^k) = (e_N, \ldots, e_k, e_{k-1}, \ldots, e_0)$.

B.1 A contraction lemma

We denote by $Conv(A)$ the convex hull of the set $A \subset \mathbb{R}$, and by $\mu_{\mathcal{L}}$ the Lebesgue measure on $\mathbb{R}$.

Lemma 2 (Contraction lemma) *Assume that $A \subset [-\bar{a}, a^{max}]$ and that the conditions of Proposition 2 are fulfilled. Let $B = \left\{ g_{a'}^{N+1}(a, (\hat{e}^{N,\bar{N}}, e^{\bar{N}})) | \hat{e}^{N,\bar{N}} \in \mathcal{E}^{N+1-\bar{N}}, a \in A \right\}$ for any $e^{\bar{N}} \in \mathcal{E}^{\bar{N}}$. We then have $\mu_{\mathcal{L}}\left(Conv(B)\right) \leq \kappa \times \mu_{\mathcal{L}}\left(Conv(A)\right)$.*

Proof. Since $B \subset \mathbb{R}$, we have by definition of the convex hull, $Conv(A) = [\min(A), \max(A)]$ and $Conv(B) = [\min(B), \max(B)]$. Let $a' = \max(A)$ and $a = \min(A)$, then $\mu_{\mathcal{L}}\left(Conv(A)\right) = a' - a$ and $B \subset [g_{a'}^{N+1}(a, (\hat{e}^{N,\bar{N}}, e^{\bar{N}})), g(a', (\tilde{e}^{N,\bar{N}}, e^{\bar{N}}))]$ for some $\hat{e}^{N,\bar{N}}, \tilde{e}^{N,\bar{N}} \in \mathcal{E}^{N+1-\bar{N}}$. Therefore, we obtain $\mu_{\mathcal{L}}\left(Conv(B)\right) \leq g_{a'}^{N+1}(a', (\tilde{e}^{N+1-\bar{N}}, e^{\bar{N}})) - g_{a'}^{N+1}(a, (\hat{e}^{N+1-\bar{N}}, e^{\bar{N}}))$. Applying the Lipschitz property (21) yields $\mu_{\mathcal{L}}\left(Conv(B)\right) \leq \kappa \times \mu_{\mathcal{L}}\left(Conv(A)\right)$. ■

B.2 Proof of the convergence of Γ^*_{N+1}

Let $N > 0$. Proposition 1 shows that when the transfer is Γ^*_{N+1}, there are $(E+1)^N$ possible end-of-period asset holdings denoted by $(a'_{\hat{e}^N})_{\hat{e}^N \in \mathcal{E}^N}$. Let $A_{N,-1}$ be the set of all possible end-of-period asset holdings in the previous period. We define:

$$A_N^{(N)}(e^N) = \{a'_{\hat{e}^N} \in A_{N,-1} | e^N \succeq \hat{e}^N\}, \text{ for } e^N \in \mathcal{E}^N, \tag{47}$$

as the set of all possible beginning-of-period and before-transfer asset holdings of agents with current history e^N. In other words, it is the set of all possible previous-period wealth levels of agents with current history e^N. Since the after-transfer wealth level $\hat{a}_{e^N}$ of (45) is an average of before-transfer wealth levels $a'_{\hat{e}^N}$, we have $\hat{a}_{e^N} \in Conv\left(A_N^{(N)}(e^N)\right)$.

We define as $\pi(e^N) = \{\hat{e}^N | e^N \succeq \hat{e}^N\}$ the set of possible predecessors of e^N. We rewrite (47) as: $A_N^{(N)}(e^N) = \{a'_{\hat{e}^N} \in A_{N,-1} | \hat{e}^N \in \pi(e^N)\}$. For any $a'_{\hat{e}^N} \in A_N^{(N)}(e^N)$, there exists $\tilde{e}^N \in \mathcal{E}^N$ such that $\hat{e}^N \succeq \tilde{e}^N$ and $a'_{\hat{e}^N} = g_{a'}^{N+1}(a_{\tilde{e}^N}, (\tilde{e}^N, \hat{e}_0^N)) = g_{a'}^{N+1}(a_{\tilde{e}^N}, (\tilde{e}^N, e_1^N))$ – since $e_1^N = \hat{e}_0^N$. In other words, $a'_{\hat{e}^N}$ is the optimal choice of an agent who, in the previous period, had the N-history $\tilde{e}^N$, which is thus a possible past of e^N. Using the notation $\pi^2 = \pi \circ \pi$, $\tilde{e}^N \in \pi^2(e^N)$, we can define:

$$A_{N-1}^{(N)}(e^N) = \{a'_{\tilde{e}^N} \in A_{N,-2} | \tilde{e}^N \in \pi^2(e^N) \text{ and } g_{a'}^{N+1}(a'_{\tilde{e}^N}, (\tilde{e}^N, e_1^N)) \in A_N^{(N)}(e^N)\},$$

which is the set of all possible end-of-period asset holdings two periods ago of agents with current history e^N. Similarly, we define for any $0 < k < N$:

$$A^{(N)}_{N-k}(e^N) = \{a'_{\tilde{e}^N} \in A_{N,-k-1} | \tilde{e}^N \in \pi^{k+1}(e^N) \text{ and } g^{N+1}_{a'}(a'_{\tilde{e}^N}, (\tilde{e}^N, e^N_k)) \in A^{(N)}_{N-k+1}(e^N)\},$$

which allows us to construct a sequence of sets $(A^{(N)}_{N-k})_{k=0,\ldots,N}$. In other words, $A^{(N)}_{N-k}(e^N)$ is the set of all possible end-of-period asset holdings k periods ago of agents with current history e^N. Iterating backward to construct those sets, we thus go back in time to construct sets of possible wealth levels (instead of histories). In the previous notation, π^{k+1} denotes $\pi \circ \ldots \circ \pi$ ($k+1$ times). Note that we could equivalently define $A^{(N)}_{N-k+1}(e^N)$ as:

$$A^{(N)}_{N-k+1}(e^N) = \{g^{N+1}_{a'}(a'_{\tilde{e}^N}, (\tilde{e}^N, e^N_k)) | \tilde{e}^N \in \pi^{k+1}(e^N) \text{ and } a'_{\tilde{e}^N} \in A^{(N)}_{N-k}(e^N)\}.$$

In other words, $A^{(N)}_{N-k+1}(e^N)$ is the set of successors (with relevant histories) of agents with wealth levels in $A^{(N)}_{N-k}(e^N)$. If $1 \leq k \leq N - \bar{N}$, we deduce, applying Lemma (2), $\mu_{\mathcal{L}}\left(Conv\left(A^{(N)}_{N-k+1}(e^N)\right)\right) \leq \kappa \mu_{\mathcal{L}}\left(Conv(A^{(N)}_{N-k}(e^N))\right)$, and iterating forward:

$$\mu_{\mathcal{L}}\left(Conv(A^{(N)}_{N}(e^N))\right) \leq \kappa^{N-\bar{N}} \mu_{\mathcal{L}}\left(Conv(A^{(N)}_{\bar{N}}(e^N))\right). \tag{48}$$

Since a^{max} ($-\bar{a}$) is by definition the highest (lowest) wealth level, $A_N \subset [-\bar{a}, a^{max}]$ and for all k, $A^{(N)}_{N-k}(e^N) \subset A_N \subset [-\bar{a}, a^{max}]$. This implies that $\mu_{\mathcal{L}}\left(Conv(A^{(N)}_{\bar{N}}(e^N))\right) \leq a^{max} + \bar{a}$. Second we have shown that $\hat{a}_{e^N}, a_{\hat{e}^N} \in Conv\left(A^{(N)}_{N}(e^N)\right)$ for any $\hat{e}^N \in \pi(e^N)$, meaning that $|\hat{a}_{e^N} - a_{\hat{e}^N}| \leq \mu_{\mathcal{L}}\left(Conv\left(A^{(N)}_{N}(e^N)\right)\right)$. As a result, we have from equation (48):$|\hat{a}_{e^N} - a_{\hat{e}^N}| \leq \kappa^{N-\bar{N}}(a^{max} + \bar{a})$, which can be made arbitrarily small ($0 < \kappa < 1$), when N increases. We deduce from (46) that $\lim_{N\to\infty} \sup_{e^{N+1}\in\mathcal{E}^{N+1}} |\Gamma^*_{N+1}(e^{N+1})| = 0$.

C Proof of Lemma 1

If we denote by $\beta^t m^t(s^t) S_{t,e^N} \lambda_{t,e^N}$ the Lagrange multiplier of the Euler equation for island e^N at date t, the objective of the Ramsey problem (22)–(29) can be rewritten as:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N}) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \tag{49}$$
$$\times \left(U_c(c_{t,e^N}, l_{t,e^N}) - \nu_{t,e^N} - \beta \mathbb{E}_t \left[\sum_{\hat{e}^N \in \mathcal{E}^N} \Pi_{t+1,e^N,\hat{e}^N} U_c(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N})(1 + r_{t+1}) \right] \right)$$

With $\lambda_{t,e^N} \nu_{t,e^N} = 0$ and definition (30) of Λ_{t,e^N}, (49) yields (after some manipulations) the objective in (31), which is maximized subject to constraints (23)–(29), with the exception of (25).

D First-order conditions of the Ramsey problem

Let $\beta^t m^t(s^t)\mu_t$ be the Lagrange multiplier of the government budget constraint (23). The Lagrangian is:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) + U_c(c_{t,e^N}, l_{t,e^N}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) \Big) \tag{50}$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \mu_t \beta^t \left(G_t + B_{t-1} + r_t A_{t-1} + w_t L_t - B_t - F(A_{t-1} - B_{t-1}, L_t, s_{t-1}) \right),$$

where $c_{t,e^N} = w_t \theta_{e_t^N} l_{t,e^N} + \delta 1_{e_0^N=0} + (1+r_t)\tilde{a}_{t,e^N} - a_{t,e^N}$, for all $e^N \in \mathcal{E}^N$, and $l_{t,e^N} = (\chi w_t \theta_{e^N})^{\varphi} 1_{e_0^N>0} + \delta 1_{e_0^N=0}$ (using (7), (24), and (26)).

Derivative with respect to B_t. We obtain the Euler equation for μ_t:

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(F_K(A_t - B_t, L_{t+1}) + 1 \right) \right]. \tag{51}$$

Derivative with respect to r_t. Computing the derivative of the Lagrangian (50) with respect to r_t yields

$$\frac{\partial \mathcal{L}}{\partial r_t} = -\mu_t \beta^t A_{t-1} + \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N}$$
$$+ \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \frac{\partial c_{t,e^N}}{\partial r_t} \Big(U_c(c_{t,e^N}, l_{t,e^N}) + U_{cc}(c_{t,e^N}, l_{t,e^N}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) \Big).$$

Noting that $\frac{\partial c_{t,e^N}}{\partial r_t} = \sum_{\hat{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}$, we deduce:

$$\mu_t A_{t-1} = \sum_{e \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N} + \sum_{e \in \mathcal{E}^N} \left(\sum_{\hat{e}^N \in \mathcal{E}^N} S_{t-1,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N} \right) \psi_{t,e_t^N}(c_{t,e^N}). \tag{52}$$

Derivative with respect to a_{t,e^N}. For all $e^N \in \mathcal{E}^N \setminus \mathcal{C}_t$, this yields:

$$\psi_{t,e^N} = \beta \mathbb{E}_t \sum_{\tilde{e}^N \in \mathcal{E}^N} (1+r_{t+1}) \Pi_{t+1,e^N,\tilde{e}^N} \psi_{t+1,e^N} - \beta \mathbb{E}_t \mu_{t+1}(r_{t+1} - \tilde{r}_{t+1}). \tag{53}$$

Combining (39) with (53) yields:

$$\forall e^N \in \mathcal{E}^N \setminus \mathcal{C}_t,\ \mu_t - \psi_{t,e^N} = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \in \mathcal{E}^N} (1+r_{t+1}) \Pi_{t+1,e^N,\tilde{e}^N} \Big(\mu_{t+1} - \psi_{t+1,\tilde{e}^N} \Big) \right]. \tag{54}$$

Derivative with respect to w_t. We obtain:

$$\mu_t L_t \left(1 + \frac{\varphi}{w_t} \left(w_t - F_L(K_{t-1}, L_t, s_{t-1}) \right) \right) = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \theta_{e^N} l_{t,e^N} \psi_{t,e^N}. \tag{55}$$

Using $w_t - F_L(K_{t-1}, L_t, s_{t-1}) = w_t - \tilde{w}_t = -\frac{\tau_t^L}{1-\tau_t^L}\tilde{w}_t$, equation (55) becomes: $\mu_t L_t \left(1 - \varphi\frac{\tau_t^L}{1-\tau_t^L}\right) = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N}\theta_{e^N} l_{t,e^N}\psi_{t,e^N}$. Since $L_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N}\theta_{e^N} l_{t,e^N}$, we deduce:

$$\sum_{e^N \in \mathcal{E}^N} \frac{S_{t,e^N} l_{t,e^N}\theta_{e^N}}{L_t}\left(\mu_t - \psi_{t,e^N}\right) = \mu_t\varphi\frac{\tau_t^L}{1-\tau_t^L}. \tag{56}$$

E Proof of Proposition 3

First-order conditions (39) and (56) immediately imply (40) at the steady-state. We now sum individual consumption Euler equations (25) for all $e^N \in \mathcal{E}^N \setminus \mathcal{C}$ (i.e., when $\nu_{e^N} = 0$):

$$\sum_{e^N \in \mathcal{E}^N \setminus \mathcal{C}} S_{e^N} U_c(c_{e^N}, l_{e^N}) = \beta(1 + (1-\tau^k)\tilde{r})\left[\sum_{\tilde{e}^N \in \mathcal{E}^N}\sum_{e^N \in \mathcal{E}^N \setminus \mathcal{C}} S_{e^N}\Pi_{e^N,\tilde{e}^N} U_c(c_{\tilde{e}^N}, l_{\tilde{e}^N})\right].$$

We now split the sum as $\sum_{e^N \in \mathcal{E}^N \setminus \mathcal{C}} = \sum_{e^N \in \mathcal{E}^N} - \sum_{e^N \in \mathcal{C}}$. We finally obtain:

$$\begin{aligned}\beta\tau^k\tilde{r}\sum_{e^N \in \mathcal{E}^N} S_{e^N} U_c(c_{e^N}, l_{e^N}) &= \sum_{e^N \in \mathcal{C}} S_{e^N} \\ &\times\left[U_c(c_{e^N}, l_{e^N}) - \beta(1 + (1-\tau^k)\tilde{r})\left[\sum_{\tilde{e}^N \in \mathcal{E}^N}\Pi_{e^N,\tilde{e}^N} U_c(c_{\tilde{e}^N}, l_{\tilde{e}^N})\right]\right],\end{aligned}$$

where we recognize the "Euler inequality" for constrained agents in the right-hand side. Using equation (25) and $\beta\tilde{r} = 1 - \beta$, we obtain equation (41).

F First-order conditions in the CM economy

We first transform the problem in which all agents belong to the same island into a standard representative agent model. This yields the complete-market outcome if agents have the same wealth in period 0 (in the general case, the ratio of marginal utilities across agents would be constant but depends on initial wealth). Denote by $\mathcal{E}_u^N$ the set of N-period histories for which households are currently unemployed (i.e., of the form $(e^{N-1}, 0) \in \mathcal{E}^N$) and earn the fixed amount δ. Denote by $\mathcal{E}_e^N$ the complement of $\mathcal{E}_u^N$ in $\mathcal{E}^N$. In the case when all agents belong to the same island, the problem faced by the family head is:

$$\max_{\left(A_t, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}} \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t\left[\sum_{e^N \in \mathcal{E}^N} S_{t,e^N}\xi_{e^N} U\left(c_{t,e^N}, l_{t,e^N}\right)\right],$$

$$\sum_{e^N \in \mathcal{E}^N} S_{t,k}c_{t,k} + A_t = w_t \sum_{e^N \in \mathcal{E}_e^N} S_{t,e^N}\theta_{e^N} l_{t,e^N} + \sum_{e^N \in \mathcal{E}_u^N} S_{t,e^N}\delta + (1 + r_t)A_{t-1},$$

$$l_{t,e^N} = \delta, \text{ if } e^N \in \mathcal{E}_u^N,$$

$$A_t \geq -\bar{a}.$$

Consider agents E^N, who are the agents with the highest productivity for the last N periods and whose history is $\{E, \ldots, E\}$. We denote by $S_t^{(0)}$, $c_t^{(0)}$, and $l_t^{(0)}$ the population size, consumption, and labor supply of these agents. We can re-write all equations as a function of $c_t^{(0)}$ and $l_t^{(0)}$. For instance, the first-order condition for labor is, for all $e^N \in \mathcal{E}_e^N$, $l_{t,e^N}^{(0)} = \left(\frac{\theta_{e^N}}{\theta_E}\right)^{\varphi} l_t^{(0)}$, from which we deduce the total labor supply:

$$L_t = \sum_{e^N \in \mathcal{E}_e^N} S_{t,e^N} \theta_{e^N} l_{t,e^N}^{(0)} = S_t^{(0)} \bar{\theta}_t l_t^{(0)},$$

$$\text{where: } \bar{\theta}_t \equiv \theta_E \sum_{e^N \in \mathcal{E}_e^N} \frac{S_{t,e^N}}{S_t^{(0)}} \left(\frac{\theta_{e^N}}{\theta_E}\right)^{1+\varphi}.$$

First-order conditions for consumption imply that marginal utilities are equalized. Hence for all $e^N \in \mathcal{E}^N$, $\xi_{e^N} U_c\left(c_{t,e^N}^{(0)}, l_{t,e^N}^{(0)}\right) = \xi_{E^N} U_c\left(c_t^{(0)}, l_t^{(0)}\right)$. We can now express the family head's program as a function of the consumption and labor choice for E^N-agents, from which we can deduce the consumption and labor choices of all other agents (using the two previous equalities). First-order conditions of the family head are:

$$\bar{\xi}_t c_t^{(0)} + A_t^{(0)} = w_t S_{t,E^N} \bar{\theta}_t l_t^{(0)} + (1 - \bar{n}_t)\,\delta + (1 + r_t^{(0)}) A_{t-1}^{(0)} \tag{57}$$
$$- \frac{1}{\chi} \frac{1}{1 + 1/\varphi} \left[\left(\frac{S_{t,E^N}}{\theta_E} \bar{\theta}_t - \bar{\xi}_t \right) \left(l_t^{(0)} \right)^{1+1/\varphi} + (1 - \bar{n}_t)\, \delta^{1+1/\varphi} \right]$$
$$l_t^{(0)} = (\chi w_t \theta_E)^{\varphi}, \tag{58}$$
$$\bar{\xi}_t U_c(c_t^{(0)}, l_t^{(0)}) \geq \beta E\left[(1 + r_{t+1}^{(0)}) \bar{\xi}_{t+1} U_c(c_{t+1}^{(0)}, l_{t+1}^{(0)}) \right] \tag{59}$$
$$A_t^{(0)} \geq -\bar{a} \tag{60}$$

where $1 - \bar{n}_t = \sum_{e^N \in \mathcal{E}_e^N} S_{t,k}$ is the measure of employed households and where $\bar{\xi}_t \equiv \left(\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(\frac{\xi_{E^N}}{\xi_{e^N}} \right)^{-\frac{1}{\sigma}} \right)$ is a parameter aggregating preference heterogeneity. Next, using the first-order conditions of the family head, we can write the Ramsey problem as:

$$\max_{\left(r_{t+1}^{(0)}, w_t^{(0)}, B_t^{(0)}, A_{t,}^{(0)}, c_t^{(0)}, l_t^{(0)}\right)_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \bar{\xi}_t U(c_t^{(0)}, l_t^{(0)}),$$

subject to the first-order conditions of the family head (57)–(60), as well as to the government budget constraint and the market clearing condition, which can be expressed as:

$$B_t^{(0)} + F(K_{t-1}^{(0)}, L_t^{(0)}, z_t) \geq G_t + (1 + r_t^{(0)}) B_{t-1}^{(0)} + r_t^{(0)} K_{t-1}^{(0)} + w_t^{(0)} L_t^{(0)},$$
$$L_t^{(0)} = S_{t,E^N} \bar{\theta}_t l_t^{(0)}, \; K_t^{(0)} = A_t^{(0)} - B_t^{(0)}.$$

Deriving first-order conditions of the Ramsey problem (with the same notation as in the main text), we obtain:

$$\mu_t^{(0)} = \beta \mathbb{E}_t \left[\mu_{t+1}^{(0)} \left(1 + \tilde{r}_{t+1}^{(0)} \right) \right],$$

$$\mu_t^{(0)} - \psi_t^{(0)} = \beta(1 + r_{t+1}^{(0)}) \mathbb{E}_t \left[\mu_{t+1}^{(0)} - \psi_{t+1}^{(0)} \right] + \nu_t^{(0)},$$

$$\mu_t^{(0)} - \psi_t^{(0)} = \varphi \mu_t^{(0)} \frac{\tau_t^{L,(0)}}{1 - \tau_t^{L,(0)}},$$

$$\mu_t^{(0)} - \psi_t^{(0)} = \frac{\bar{\xi}_t U_c(c_t^{(0)}, l_t^{(0)}) \lambda_{t-1}^{(0)}}{A_{t-1}^{(0)}},$$

where $\psi_t^{(0)} \equiv U_c(c_t^{(0)}, l_t^{(0)}) + U_{cc}(c_t^{(0)}, l_t^{(0)}) \left((1 + r_t^{(0)}) \lambda_{t-1}^{(0)} - \lambda_t^{(0)} \right)$.

Steady state. It is easy to check that the interior steady-state equilibrium has the following properties: $\tau^{K,(0)} = 0$, $A^{(0)} = -\bar{a}$, together with $\nu^{(0)} = 0$. Moreover, we have $\tau^{L,(0)}, \lambda^{(0)} > 0$ if and only if $\mu^{(0)} > \psi^{(0)}$.

G Algorithm for matching distributions

We explain how to match a steady-state wealth distribution using $(\xi_{e^N})_{e^N \in \mathcal{E}^N}$, for a given N. Transition probabilities $(\Pi_{e^N, \tilde{e}^N})_{e^N, \tilde{e}^N \in \mathcal{E}^N}$ and population sizes (S_{e^N}) are constant. We first simulate a Bewley model with the prices given in the text and derive end-of-period wealth levels $(a_{e^N})_{e^N \in \mathcal{E}^N}$ using the policy rules. We also obtain the set $\mathcal{C}$ of credit-constrained histories. This set has to be non-empty for the following algorithm to work. In our simulations, it is the case for not too short truncations. Other steps are:

1. Derive the beginning-of-period wealth level from (7). Then, derive the consumption for each history, c_{e^N}, using the labor supply (13) and the budget constraints (9).

2. Use consumption levels and labor choices to derive the values $U_c(c_{e^N}, l_{e^N})$.

3. Set parameters ξ_{e^N} for constrained histories $e^N \in \mathcal{C}$ to ξ_c. Then use (12) to derive the following $\xi_{e^N} U_c(c_{e^N}, l_{e^N}) = \beta \left[\sum_{\tilde{e}^N \succeq e^N} \Pi_{e^N, \tilde{e}^N} \xi_{\tilde{e}^N} U_c(c_{\tilde{e}^N}, l_{\tilde{e}^N})(1 + r) \right]$ for all unconstrained histories $e^N \in \mathcal{E}^N \setminus \mathcal{C}$.

4. Solve the previous equations, which form a linear system in ξ_{e^N} for $e^N \in \mathcal{E}^N \setminus \mathcal{C}$, as a function of ξ_c. Then iterate over ξ_c until $\sum_{e^N \in \mathcal{E}^N} S_{e^N} \xi_{e^N} = 1$.

H Algorithm to find the steady state

1. We guess the set $\mathcal{C}$ of islands that are credit-constrained. We choose a post-tax interest rate r and a post-tax wage rate w. Then:

(a) We compute the labor supply on each island e^N, using agents' first-order conditions (26). We then deduce the aggregate labor L. We compute the aggregate capital K with $F_K(K, L) = \beta^{-1} - 1$.

(b) We determine individual consumption levels and asset holdings using equations (24), (25), and (7). We deduce a corresponding value for public spending, given by $G = F(K, L) - rA - wL - T$.

(c) We set a value of μ. We set values ψ_{e^N}, $e^N \in \mathcal{C}$ (for credit-constrained islands). Using (54), we then solve for ψ_{e^N}, $e^N \in \mathcal{E}^N \setminus \mathcal{C}$ (unconstrained islands). We then obtain λ_{e^N}, $e^N \in \mathcal{E}^N$ using (33) defining ψ_{e^N}. We finally iterate on ψ_{e^N}, $e^N \in \mathcal{C}$, until we have $\lambda_{e^N} = 0$ for $e^N \in \mathcal{C}$ (constrained islands). We iterate on μ until equation (38) holds at the steady-state.

(d) We iterate on w until (56) holds at the steady-state.

2. We iterate on r until G/Y matches its target. We finally verify that the Euler inequalities are strict for islands $e^N \in \mathcal{C}$ to check that the set $\mathcal{C}$ of constrained islands is correct. Otherwise, we iterate on $\mathcal{C}$.