Sovereign Default and Liquidity: The Case for a World Safe Asset*

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Abstract

This paper presents a normative study of a world financial market in which sovereign countries can default on their debt. In our model, the world interest rate is determined by countries' net savings. The amount of safe assets is endogenous and determines international risk sharing. First, we show that non-trivial multiple equilibria naturally arise, due to the endogeneity of the interest rate. The quantity of safe assets is thus not uniquely determined. Second, even the market equilibrium with the highest welfare is not constrained-efficient because the market incentives for defaulting are too high. Third, we prove that a world fund issuing a safe asset increases aggregate welfare. Its relationship with the IMF's Special Drawing Rights is discussed.

1 Introduction

The global economy exhibits two related features. The first is the pervasiveness of sovereign default. The largest default in history (by present value) was the 2012 Greek debt restructuring, which covered more than €200 bn of privately held debt (Tomz and Wright 2013). Current debates regarding a larger restructuring of Greek debt or the default of another developed country indicate that sovereign default may reach another order of magnitude in the near future. Even if the default risk fails to materialize, the fact that it is discussed is a clear indication that

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the perceived safeness of certain countries' public debt has deteriorated. The second feature affecting the global economy is the apparent shortage of safe assets. Caballero and Farhi (2017), Barro and Mollerus (2016), and Hall (2016), among others, observe the downward trend in the return on US public debt and discuss the effects of a shortage of safe assets.[1] These two facts are obviously related, as sovereign default almost mechanically reduces the quantity of safe assets. In addition, the quantity of safe assets affects real interest rate levels and the default probability through its effect on both the incentives to accumulate assets and the opportunity cost of default.

The goal of this paper is to investigate the interactions between the quantity of safe assets and sovereign default in general equilibrium. We are interested in both positive and normative questions. What are the determinants of the quantity of safe assets when sovereigns can choose to strategically default? Does the market economy generate too many defaults as a general equilibrium outcome? Should a world institution issue a safe asset? If so, how much should it issue? This last question was, for instance, discussed in policy circles at the IMF before the decision to issue Special Drawing Rights.

To help answer these questions, this paper presents a general equilibrium model of default where sovereigns can borrow or lend on international markets, and possibly default on their debt, in order to smooth idiosyncratic income shocks. Elaborating on the Eaton and Gersovitz (1981) literature, the basic friction we consider is the lack of a complete insurance market for country-specific risk. Countries can only issue non-contingent claims, on which they can default. Equilibrium default can occur when the debt burden is high in bad times and when the opportunity cost of default is low. Following the work of Aguiar and Gopinath (2006) and Arellano (2008), among others, this framework has become a benchmark for studying sovereign default (see the recent paper by Amador and Aguiar 2015). In this literature, the authors assume that the riskless real interest rate is exogenous, which enables them to investigate the amount of debt and the default decision in rich environments (see the literature review below). Our contribution is to endogenize the world interest rate by developing a tractable general equilibrium model and to derive welfare analysis.

We first model the world economy as a continuum of countries borrowing and lending to one other to smooth idiosyncratic shocks. Countries can default on their debt depending on

[1] See also Gorton, Lewellen, and Metrick (2012) for a measure of the safe asset share in the US economy.

the endogenous intertemporal cost of default. When countries default, they are temporarily excluded from the financial markets. The world interest rate is determined by the international financial market clearing system. Equilibrium in this economy is characterized by the default policy, world wealth distribution, and world interest rates. An alternative way of considering this economy is to think about a Bewley economy where agents face no exogenous credit constraint but may default if they find it optimal to do so. Surprisingly, the constrained-efficiency of market economies and of optimal policies in these environments has yet to be studied. To do so, we use the strategy of Davila, Hong, Krusell, and Rios-Rull (2012), developed in incomplete-market economies without default, in order to derive the constrained-efficient allocation. We then analyze the properties of the market economy, in particular welfare and the quantity of safe assets determining international risk sharing. We derive three main results.

The first result is that the interaction between incomplete markets and default generates multiple equilibria, which differ in terms of the quantity of safe assets and interest rates. When the interest rate is high, the incentives to save for self-insurance are high. As countries save more, there are fewer defaults because the opportunity cost of defaulting is high. Participating in financial markets is highly valued as risk sharing is important. A high supply of safe assets is consistent with a low default risk. Conversely, when a smaller supply of safe assets is available for self-insurance, there is a higher default rate in general equilibrium, as the relative gain of participating in financial markets is low. The multiple equilibria can be ranked according to their aggregate welfare. An equilibrium with fewer defaults unambiguously corresponds to higher welfare, compared to an equilibrium with a higher number of defaults. The multiplicity of equilibria thus crucially depends on the endogeneity of the interest rate.

Second, we show that the market equilibrium with the highest welfare is still constrained-inefficient. The reason being that when markets are incomplete, prices do not convey the right incentives to save. In other words, there is a pecuniary externality, implying a lack of risk sharing and a shortage of safe assets in the market economy. Sovereigns do not properly internalize the social benefits of their debt as a safe asset for other countries in their net saving and default decisions, as markets are incomplete.

Third, welfare is increased via the introduction of an international fund. This fund issues interest-bearing assets that are financed via the voluntary contributions of member states. There is an equilibrium where all countries are either excluded from the financial markets because they have defaulted or voluntarily contribute to the fund. In this case, there is full risk sharing among the countries participating in financial markets, but default still occurs in general equilibrium. We calibrate the model using data on sovereign default and "disaster events" to reproduce both the default probability and income fluctuations. The optimal quantity of assets issued by the fund represents around 13% of world GDP. Countries' average contributions are equal to 0.5% of their own GDP.

As a final step, we verify that these results are robust to the introduction of a large country – with positive mass – internalizing the effect of its saving decisions on the world financial market. Following Farhi and Maggiori (2018), we call this country the *Hegemon*. We prove that the main results hold, even in the presence of the Hegemon. There are too many sovereign defaults in the general equilibrium and there are insufficient safe assets. The introduction of an international fund can restore an aggregate welfare equal to that corresponding to the constrained-efficient allocation.

The fund and its asset supply are obviously reminiscent of the IMF Special Drawing Rights (SDRs). These SDRs were issued in the early 1970s after a global discussion about the scarcity of safe stores of value (see Williamson 2009 for a short history). SDRs are interest-bearing assets, whose interest rate is determined weekly as the average interest rate on the money markets for a basket of currencies.[2] There are nevertheless two main differences between the assets issued by the fund in our model and SDRs. First, the interest rate on the assets issued by the fund in our model should be linked to the interest paid by governments issuing safe assets rather than the interest on money markets, as is the case for SDRs. Second, the outstanding amounts differ substantially. For SDRs, the outstanding amount was smaller than 0.3% of world GDP in 2016, while, with our calibration, the optimal issuance of world safe assets should correspond to 13% of world GDP.

Literature review. This model provides positive and normative implications for sovereign default in general equilibrium. It follows the tradition of Eaton and Gersovitz (1981) based on incomplete insurance markets, which have been shown to be quantitatively relevant (Aguiar and Gopinath 2006, Arellano 2008, among others, as well as the recent survey of Amador and Aguiar

[2]See the IMF website for a precise definition and SDR interest rate values: `http://www.imf.org/external/np/fin/data/sdr_ir.aspx`.

2015). The same tools (incomplete insurance markets and strategic default) have been used to study household default in general equilibrium. Chatterjee, Cordae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007) use this model to compare the welfare effect of different legal frameworks on household default. Instead, we provide a model to characterize the constrained efficient equilibria using the methodology of Davila, Hong, Krusell, and Rios-Rull (2012) and provide a mechanism to decentralize it.

Our analysis stresses the role of the quantity of assets required to self-insure against idiosyncratic shocks. It is thus related to the recent literature on safe assets, in particular the literature focusing on the quantity of the risk-adjusted store of value provided by the market economy. Authors (Gorton, Lewellen, and Metrick 2012, Barro and Mollerus 2016, Hall 2016, and Caballero and Farhi 2017, among others) have studied the implications of a possible lack of safe assets. Another large body of literature investigates the determinants of the quantity of world safe assets and of the interest rate (Mendoza, Quadrini, and Rios-Rull 2009, Azzimonti, de Francesco, and Quadrini 2014) and the decisions of sovereigns to strategically choose their level of public debt with roll-over risk (He, Krishnamurty, and Milbradt 2018). Farhi and Maggiori (2018) propose a model of the international monetary system and study various market structures and frictions. We instead identify the distortions on the world financial markets in the absence of these strategic behaviors.

The source of multiple equilibria is new to the best of our knowledge, although other mechanisms generating equilibrium multiplicity have been discussed in the literature. First, a large body of literature has emphasized roll-over risk, following Calvo (2018) or Cole and Kehoe (2000). Recent references include Lorenzoni and Werning (2013) and He, Krishnamurty, and Milbradt (2018). In our model, this type of risk does not exist. Recently, Arellano, Bai, and Lizarazo (2018) have proved the presence of multiple equilibria generated by negative wealth shocks on risk-adverse lenders. Our mechanism for equilibrium multiplicity is different. It is based on the endogeneity of the quantity of safe assets, which affects both the cost of self-insurance and the incentives to default. Indeed, when the interest rate is exogenous, the equilibrium is unique in our Eaton-Gersovitz environment as shown by Auclert and Rognlie (2016).[3]

Section 2 presents the environment. Section 3 solves the model for the market equilibrium

[3] Finally, there is a different literature stream on default in general equilibrium with complete insurance markets, where default generates utility costs. See, for instance, Zame (1993) and Dubey, Geanakoplos, and Shubik (2005), among others.

and studies equilibrium multiplicity. Section 4 presents the constrained-efficient allocation. Section 5 shows that this allocation can be decentralized with a fund. Section 6 presents a numerical application. Section 7 introduces a large country internalizing the effect of its saving decision on the world financial markets. Section 8 concludes.

2 Environment

2.1 Set-up

Time is discrete $t = 1, \ldots, \infty$. We first model the world economy as a continuum of small open economies. Countries are distributed in each period t according to a uniform distribution G over a segment I of length 1, without mass points.[4] All countries are thus small and have no market power in the world financial market. In Section 7 below, we introduce a large country with financial market power, to verify the generality of our results.

Each economy has a risky production technology and a benevolent government maximizes utility on behalf of a unit mass of identical consumers. Countries face idiosyncratic production shocks and there is no aggregate shock at the world level.

Each country has identical, additive, and time-separable preferences over streams of consumption $(c_t)_{t\geq 0}$ and labor supply $(l_t)_{t\geq 0}$. The period utility function over consumption c and labor supply l is $u(c) - l$, where the consumption utility function u is increasing, twice continuously differentiable, and concave. The micro-foundation for this linear disutility of labor is the existence of both indivisible labor and a complete market within the country as shown by Hansen (1985) and Rogerson (1988).[5] In each country, the government maximizes the intertemporal welfare $\sum_{t=0}^{\infty} \beta^t (u(c_t) - l_t)$, where $\beta \in (0, 1)$ is the constant discount factor.

Countries face a productivity risk that can be neither avoided nor insured. The productivity status of a given country can be in one of two states, which are described as *productive* (state p) and *unproductive* (state u). When a country is productive, it has access to a linear production technology, which transforms l units of labor into l units of final goods. The country's labor

[4]This representation of the world economies follows Clarida (1990) or Bai and Zhang (2010). There is a body of literature examining the applicability of the law of large numbers in continuum economies – see Feldman and Gilles (1985) and Green (1994), among others. Here, we simply assume that the law of large numbers applies.

[5]The utility function is used by Lagos and Wright (2005) in a matching environment and by Scheinkman and Weiss (1986) and Challe, LeGrand, and Ragot (2013), among others, in an incomplete-market environment without default. It simplifies the analysis, as shown below.

supply can, in addition, be freely adjusted in every period. When a country is unproductive, it is restricted to supplying an amount $\bar{l} < 1$ of labor. This restriction implies that the marginal utility of consumption for a country in a productive state is smaller than that of a country in an unproductive state.

The productivity status follows a first-order Markov chain with the transition matrix $\Pi = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\rho & \rho \end{bmatrix} \in (0,1)^4$, where the probability $1-\alpha$ is the probability of switching from state p in the current period to state u in the next one, for example.

2.2 Financial markets and default

As is standard in the literature on sovereign default, following Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), international financial markets are assumed to be plagued by two frictions. First, markets are incomplete for the country-specific production shock. Second, countries can default on their debt, but at the cost of being temporarily excluded from the financial markets. We assume that the current productivity status and the asset volume for each country are observable when debt is traded on international markets. If a country defaults on its debt, it is excluded from international markets and has to live in a state of autarky before potentially rejoining the international financial markets. Excluded countries are affected by a productivity shock.

To make the model empirically relevant, we follow Arellano (2008) and introduce an additional cost of default. Countries in autarky only have access to a less efficient production technology, through which 1 unit of labor is transformed into only $\varphi \in (0,1]$ units of consumption. Excluded productive countries have the probability $\theta \in (0,1)$ of reentering the financial markets in every period, while excluded unproductive agents cannot reenter the financial markets. When reentering the financial markets, countries are endowed with zero financial wealth.

Debt is traded through a unit mass of competing risk-neutral financial intermediaries. Saving countries lend to intermediaries and borrowing countries borrow from them. Financial intermediaries diversify their risk across countries and act as devices for pooling idiosyncratic risk. Since there is no aggregate risk, financial intermediaries can charge a credit risk premium to borrowers according to their default probability and savers can save in a safe asset.

We denote by $q(B', s)$ the price of a claim on one unit of a next period good for a country that chooses an amount of asset B' and that has the production status $s = \{p, u\}$. If $B' > 0$, the country is saving in a safe asset of price $q(B', s) = q$. The real interest rate on the safe asset is simply $1 + r \equiv \frac{1}{q}$. If $B' < 0$, the country is borrowing. We follow Arellano, Bai, and Bocola (2017) and Arellano, Bai, and Mihalache (2018) and assume that the debt repayment in cases of default is exogenous and equal to $\underline{B}$. The recovery rate is therefore endogenous and amounts to $\underline{B}/B'$, which depends on the asset choice B'. This recovery rate is introduced to be consistent with the data, but is not important for the qualitative results.

Since intermediaries are perfectly diversified, the price $q(B', s)$ can be expressed as:

$$q(B', s) = q \times \left(1 - \delta(B', s) + \delta(B', s)\frac{\underline{B}}{B'}\right),$$

where $\underline{B} < 0$ is the constant debt repayment and $\delta(B', s)$ is the default probability of a country choosing a debt B' while its current status is $s \in \{p, u\}$. Importantly, both the price of a safe asset q and the default probability $\delta(B', s)$ will be general equilibrium endogenous outcomes.

A country in state $s \in \{p, u\}$, endowed with the beginning-of-period wealth B, trading an amount of debt B', and supplying a labor quantity l, will have consumption equal to:

$$c = l + B - q(B', s) B'. \tag{1}$$

If the country is in a productive state, it can then freely adjust its labor effort l. If unproductive, the country will supply the fixed amount $l = \bar{l}$.

Since the default probability $\delta(B', s)$ is not necessarily continuous for the debt level B', the budget set may not be convex. However, although the problem is not convex, it can be written in recursive form (see Stokey and Lucas 1989, Theorem 9.4 and Auclert and Rognlie 2016 for the properties of the equilibrium structure), which allows us to simplify the exposition and to derive the necessary first-order conditions.

2.3 Welfare functions and financial market clearing

We define $V_s^o(B)$ as the value function of a country that participates in the financial markets, that starts the current period with the asset holding B, and that has the productivity status $s \in \{p, u\}$.[6] The superscript o denotes a participating country that has the *option* to default

[6]For the sake of simplicity, we follow the notations of Arellano (2008).

on B. The country then decides whether to default or repay its debts by choosing the option associated with the highest welfare. We denote by $V_s^c(B)$ the value function of a country deciding to repay the debt B, while its status is s. The superscript c indicates that the country decides to *continue* to honor its debts. Similarly, the value function $V_s^d(b)$ is the value function of a country that decides to default. The dependence in b is only designed to take into account the fixed debt repayment $\underline{B}$, in a recursive formula, as can be seen in equations (3) and (4) below. The superscript d denotes the *default* decision. The value function $V_s^o(B)$ is thus equal to the maximum between the value functions associated with debt repayment or default. Formally:

$$\forall B, \forall s \in \{p, u\}, \quad V_s^o\left(B\right) = \max\{V_s^c\left(B\right), V_s^d\left(\underline{B}\right)\}. \tag{2}$$

Let us now turn to the expression of value functions V^d and V^c. Since the probability of defaulting countries reentering the financial markets when productive with zero wealth is θ, the value function associated with default can be expressed as:

$$V_p^d(b) = \max_l \ \left(u\left(\varphi l + b\right) - l\right) + \beta\alpha\left(\theta V_p^o\left(0\right) + \left(1-\theta\right) V_p^d\left(0\right)\right) + \beta(1-\alpha)V_u^d\left(0\right), \tag{3}$$

$$V_u^d(b) = u\left(\varphi\bar{l} + b\right) - \bar{l} + \beta(1-\rho)\left(\theta V_p^o\left(0\right) + \left(1-\theta\right) V_p^d\left(0\right)\right) + \beta\rho V_u^d\left(0\right). \tag{4}$$

In these two equations, the country is excluded from the financial markets and trades no assets. The whole production φl or $\varphi\bar{l}$ of a period is consumed within the same period. The dependence in b in (3) and (4) is purely designed to distinguish between countries that have just defaulted and are paying the debt amount $\underline{B}$ and countries that were already in autarky.

When a country participates in the financial markets and chooses not to default in the current period, the value function V^c can be expressed as follows, for any debt endowment B:

$$V_p^c\left(B\right) = \max_{l,B'} \ u\left(l + B - q\left(B', s\right)B'\right) - l + \beta\left(\alpha V_p^o\left(B'\right) + (1-\alpha)V_u^o\left(B'\right)\right), \tag{5}$$

$$V_u^c\left(B\right) = \max_{B'} \ u\left(\bar{l} + B - q\left(B', s\right)B'\right) - \bar{l} + \beta\left((1-\rho)V_p^o\left(B'\right) + \rho V_u^o\left(B'\right)\right), \tag{6}$$

where we use the budget constraint (1) to substitute for the consumption expression. As the value functions V_s^c for $s = p, u$ are increasing in the wealth amount B, we can define the quantities $\overline{B}^s \in \mathbb{R} \cup \{-\infty\}$, $s = p, u$ as:

$$\overline{B}^s = \min_{B \in \mathbb{R}}\{B | V_s^c\left(B\right) \geq V_s^d\left(\underline{B}\right)\}, s = p, u. \tag{7}$$

In other words, the quantity $\overline{B}^s$ is the state-contingent threshold such that a country endowed with a wealth level above the threshold will decide not to default.

Finally, we provide the expression for financial market clearing. Since assets are in zero net supply and competition among financial intermediaries implies a zero-profit condition, the value of all net asset demands sums to zero. If we recall that countries are distributed along the segment I with the distribution G, financial market clearing can be formalized as follows:

$$\int_{i\in I} q\left(B'^i, s^i\right) B'^i G\left(di\right) = 0, \tag{8}$$

where B'^i is the asset demand of country $i \in I$ in the current period. We focus on the steady state equilibrium where the price of the safe asset q is constant. We provide a more formal definition of the equilibrium.

Definition 1 (Competitive equilibrium) *A competitive equilibrium is a price q, risk premia $(\delta(B,s))_{s=u,p,B\in\mathbb{R}}$, policy functions $(B'(B,s))_{s=u,p,B\in\mathbb{R}}$, $l(B,p)_{B\in\mathbb{R}}$, and the level of production in autarky l^d such that: 1) for a given price q and given risk premia $(\delta(B,s))_{s=u,p,B\in\mathbb{R}}$, policy functions solve the program (2)–(6); 2) the risk premia are consistent with perfect competition and full diversification for financial intermediaries; and 3) the price q is such that the financial market clears, i.e., equation (8) holds.*

3 Market equilibrium

We present the main properties of the market equilibrium. We start by the equilibrium price schedule, which is standard in this type of environment. Following Arellano (2008), among others, we know that two asset thresholds $\overline{B}^e$and $\overline{B}^u$, $\overline{B}^e < \overline{B}^u < 0$ exist such that:

$$q\left(B'\right) = \begin{cases} q & \text{if } B' \geq \overline{B}^u, \\ q\left(\alpha + (1-\alpha)\frac{B}{B'}\right) & \text{if } \overline{B}^e \leq B' < \overline{B}^u, \\ q\frac{B}{B'} & \text{if } B' < \overline{B}^e, \end{cases} \quad \text{and } q\left(B', u\right) = \begin{cases} q & \text{if } B' \geq \overline{B}^u, \\ q\left(1-\rho+\rho\frac{B}{B'}\right) & \text{if } \overline{B}^e \leq B' < \overline{B}^u, \\ q\frac{B}{B'} & \text{if } B' < \overline{B}^e. \end{cases} \tag{9}$$

Equation (9) states that the price schedule is a step function with two thresholds $\overline{B}^e$ and $\overline{B}^u$, where $\overline{B}^e < \overline{B}^u$. If a country's net saving is higher than $\overline{B}^u$, then it will not default in the next period, regardless of its idiosyncratic shock. In this case, the price of its claim is the price of a safe asset q. If the country saves less than $\overline{B}^u$ but more than a second threshold $\overline{B}^e$, it will default in cases of bad idiosyncratic shock, but not in cases of good shock. Consequently, the price of the claim is the price q multiplied by the expected payoff in the next period. This expected payoff is either 1 if the country remains productive or the recovery ratio $\frac{B}{B'}$ if the country becomes unproductive and defaults. Finally, for net savings that are strictly smaller than $\overline{B}^e$, the country defaults regardless of the next-period idiosyncratic shock, so that the price of the claim is $q\frac{B}{B'}$.

3.1 Euler equations and budget constraints

Quasi-linearity in the labor supply considerably simplifies the analysis of the equilibrium. To begin with, the first-order condition for the labor supply l of productive countries in (5) yields $u'(c_0) = 1$. As a result, we find that the consumption level of productive countries is $c_0 = u'^{-1}(1)$. Using the budget constraint (1), the value function (5) becomes:

$$V^c(B,p) = V^c(0,p) + B. \tag{10}$$

The value function of productive countries participating in the financial markets is thus linear in the beginning-of-period asset endowment. All productive non-defaulting countries therefore have the same marginal utility, which is independent of their beginning-of-period endowment.

We now prove that the equilibrium is characterized by a sequence $\{B_k\}_{k=0,\ldots,\infty}$, such that all productive countries participating in financial markets save B_0 while participating countries that are unproductive for $k \geq 1$ consecutive periods save the same amount B_k. In equilibrium, the saving (or borrowing) decision of agents thus simply depends on their idiosyncratic production history, which considerably simplifies aggregation.

We adopt a guess-and-verify strategy to simplify the exposition. Assume that countries default after being unproductive for $D \leq \infty$ consecutive periods, where the case $D = \infty$ corresponds to countries never defaulting. The first-order conditions for asset choices are:

$$q = \beta\left(\alpha + (1-\alpha)u'(c_1)\right), \tag{11}$$

$$qu'(c_k) = \beta\left(1 - \rho + \rho u'(c_{k+1})\right), \quad k = 1, \ldots, D-2, \tag{12}$$

$$qu'(c_{D-1}) \geq \beta, \tag{13}$$

where $c_k,\ k = 1, \ldots, D-1$ denotes the consumption level of a participating country that is unproductive for the k consecutive prior periods. The budget constraints (1) become:

$$B_{k-1} = c_k - \bar{l} + qB_k,\ k = 1, \ldots, D-2, \tag{14}$$

$$B_{D-2} = c_{D-1} - \bar{l} + q(1-\rho)B_{D-1} + q\rho\underline{B}, \tag{15}$$

We denote by l_{k0} the labor supply of productive countries that have been unproductive for the previous k periods and have just returned to productivity. By extension, l_{00} denotes the labor supply of productive countries that were already productive in the previous period, and l_{-10} is the labor supply of participating productive countries that were excluded from financial markets in the previous period and that have just reentered the financial markets. Recall that the beginning-of-period wealth of these countries will also be assumed to be null: $B_{-1} = 0$. The budget constraint (1) implies:

$$l_{k0} = u'^{-1}(1) - B_k + qB_0,\ k = -1, 0, \ldots, D-1, \tag{16}$$

The intuition for this simple equilibrium structure is that all productive countries choose the same consumption level $u'^{-1}(1)$ and the same asset holding B_0 due to the quasi-linearity of the utility function (and the linearity of the value function). The labor supply of productive countries l_{k0} adjusts to compensate for the difference in their beginning-of-period wealth levels. This wealth level depends on the country's productive status in the previous period and in particular on the number k of consecutive past periods of being unproductive (with $k = 0$ corresponding to a productive status and $k = -1$ to a default status).

Remark 1 (Notation) *To lighten the notation, in the remainder of the article we consistently employ the notation introduced in this section. For a variable x, x_k is the quantity associated with a country that is unproductive for $k = 1, \ldots, D-1$ periods, while x_{k0} denotes the quantity for a productive country that was unproductive for the $k = -1, 0, \ldots, D-1$ previous periods. For instance, x_{20} concerns a productive country that was unproductive in the last two previous periods. The quantity associated with a productive country that was already productive in the previous period is x_{00}, while x_{-10} refers to a productive country that was excluded. When the past status has no influence for a productive country – for instance in the case of consumption level – we simply use the notation x_0.*

3.2 Default conditions and limited-heterogeneity equilibrium

Countries will default after being unproductive for exactly D consecutive periods if the following conditions hold: (i) paying back debt dominates the default option if the countries are unproductive for less that $D-1$ consecutive periods; (ii) countries unproductive for exactly D consecutive periods would be worse off paying back their debt than defaulting and paying $\underline{B}$; (iii) productive countries that have previously been unproductive for k consecutive periods choose to pay back their contracted debt while unproductive and do not default. This implies:

$$V_u^c(B_k) \geq V_u^d(\underline{B}),\ 0 < k < D, \tag{17}$$

$$V_p^c(B_k) \geq V_p^d(\underline{B}),\ 0 < k < D, \tag{18}$$

$$V_u^d(\underline{B}) \geq V_u^c(B_D), \tag{19}$$

where the utility in cases of default depends on the debt repayment $\underline{B}$. Note that the inequality (19) implies that the value $\underline{B}$ cannot be too negative. In other words, the recovery ratio $\underline{B}/B'$ has to remain sufficiently small.

We can now express the financial market clearing condition. First, we denote by n_k the population of participating countries that are unproductive for k consecutive periods for $1 \leq k \leq D$. From transition probabilities, we can state that $n_k = \rho^k n_0$ for $k = 1, \ldots, D-1$, where n_0 denotes the mass of productive countries participating in financial markets. In the interests of conciseness, the explicit expression of these population shares is shown in Appendix A. Using these notations and substituting for equilibrium population shares and prices, the financial market equilibrium (8) can be expressed as follows:

$$\sum_{k=0}^{D-2} n_k B_k + (1-\rho) n_{D-1} B_{D-1} + \rho n_{D-1} \underline{B} = 0. \tag{20}$$

To summarize, the equilibrium is thus characterized by: 1) a number of periods D before default, with the possibility that $D = \infty$, if default never occurs; 2) consumption levels $(c_k)_{k=0,\ldots,D-1}$, asset demands $(B_k)_{k=0,\ldots,D-1}$, and labor efforts $(l_{k0})_{k=-1,0,\ldots,D-1}$ satisfying Euler equations (11)–(13) and budget constraints (14)–(16), while conditions (17)–(19) hold; 2) a price for the safe asset q, enabling the market to clear and equation (20) to hold.

As we show in our quantitative exercise in Section 6, the value D is finite for standard calibrations. The economy is thus populated by a finite number of different types of countries, where each type chooses the same consumption and wealth. More precisely, $D-1$ unproductive types participating in financial markets are characterized by the number $k = 1, \ldots, D-1$ of consecutive periods for which they are unproductive. Furthermore, $D+1$ productive types participate in financial markets and are characterized by their status in the previous period, which could be either: (i) unproductive, (ii) productive, or (iii) excluded from the financial markets. As explained above, productive countries choose the same consumption and asset holding and only differ with respect to their labor effort. In addition to these $2D$ participating types, 2 additional types of country are excluded from the financial markets, and differ according to their productive status. On the whole, the economy is populated by $2D+2$ different types of countries.

We can now compute the welfare for each type of country. Consistent with our previous notation, we denote by V_k the intertemporal welfare of a country participating in financial markets and unproductive for $k = 1, \ldots, D-1$ periods, while V_{k0} denotes the intertemporal welfare of a productive country participating in financial markets, which was productive in the $k = -1, \ldots, D-1$ previous periods. Formally, we have:

$$V_{k0} = u\left(u'^{-1}(1)\right) - l_{k0} + \beta\left(\alpha V_{00} + (1-\alpha)V_1\right), \; k = -1, \ldots, D-1, \tag{21}$$

$$V_k = u\left(c_k\right) - \bar{l} + \beta\left((1-\rho)V_{k0} + \rho V_{k+1}\right), \; k = 1, \ldots, D-1. \tag{22}$$

Using expressions (21) and (22), participation conditions (17) and (19) become $V_k \geq V_u^d(\underline{B})$ and $V_{k0} \geq V_p^d(\underline{B})$ for $k = 1, \ldots, D-1$.

The next proposition further characterizes the equilibrium structure.

Proposition 1 (Limited-heterogeneity equilibrium characterization) *The price q verifies $q \geq \beta$ and we have* either*:*

(a) $D = \infty$, $q > \beta$ *and* $\lim_{k\to\infty} B_k = -\frac{\bar{l}}{1-q}$; or

(b) $D < \infty$ *and* $V_p^c(B_{D-1}) = V_p^d(\underline{B})$.

Proposition 1 states that two types of equilibrium can exist. The first (a) is an equilibrium in which default never occurs. The second (b) is an equilibrium in which the borrowing limit B_{D-1} for unproductive countries is determined by their default incentive when becoming productive in the next period and repaying $\underline{B}$. In this equilibrium, unproductive countries borrow increasing amounts while remaining unproductive, until they reach an amount that would lead them to default in the following period if they became productive. A productive country that starts the period with a large amount of debt would have to provide a significant labor effort to repay its debt and it may thus be better off defaulting.

Although it characterizes a possible equilibrium structure, Proposition 1 is silent about the uniqueness of the possible equilibrium. Section 3.3 below proves that multiple equilibria are possible.

Finally, we will compare the multiple equilibria using a utilitarian welfare criterion. Other criteria could be used, but this one seems to be the most natural in heterogeneous-agent economies, for reasons already discussed in Aiyagari (1994), and it is thus widely used in the literature. Using the share of different types of countries, the expression of aggregate welfare W^a is simply:

$$W^a = \sum_{k=-1}^{D-1} n_{k0} V_{k0} + \sum_{k=1}^{D-1} n_k V_k \tag{23}$$
$$+ n_p^d V_p^d(0) + n_{u,1}^d V_u^d(\underline{B}) + \left(n_u^d - n_{u,1}^d\right) V_u^d(0),$$

where n_s^d $(s = p, u)$ is the total number of countries in state s that have defaulted and are in autarky, and $n_{u,1}^d$ is the number of unproductive countries that have just defaulted, and therefore reimburse $\underline{B}$.

Aggregate welfare is the sum of the intertemporal welfare of countries participating in the financial markets, whether they are productive or unproductive, and the intertemporal welfare of countries excluded from the financial markets, again whether they are productive or unproductive.

3.3 Multiple equilibria

The previous section characterized the equilibrium structure in the general case. We now explain why non-trivial multiple equilibria can occur, with different numbers of periods before default (i.e., different values of D), different amounts of liquidity, and different prices for the safe asset (and thus different values for the real interest rate). The key mechanism for multiple equilibria is the endogeneity of asset prices. We provide examples of such equilibria in the quantification exercise below and discuss here the mechanisms and welfare properties.

Countries self-insure in good times by purchasing the safe asset. The quantity of safe assets amounts to the risk-adjusted quantity of debt in the economy. When the safe asset is scarce, its price q is high and the risk-free interest rate r is low. Self-insurance is expensive as the return on savings is low and countries have little incentive to save in good times for self-insurance motives. As a result, countries' demand for the safe asset is low and consumption smoothing is poor, as can be observed from the first-order conditions of participating agents (11). When the interest rate is low, countries default more often, which reduces the supply of safe assets. Conversely, when there is abundant liquidity, the welfare gains of self-insurance are high, the interest rate is high, and countries save to self-insure and default less often. The quantity D is high in equilibrium.

This source of equilibrium multiplicity is not reliant on countries having a positive probability of reentering the financial markets ($\theta > 0$). In fact, we also consider an alternative theoretical model, in which we remove this channel. More precisely, we assume that defaulting countries are excluded from the financial markets for ever and are replaced with probability θ by new countries with zero wealth, with the same parameters. The value of the default states (see equations (3) and (4)) is exogenous and does not depend on the equilibrium. We also find multiple equilibria with multiple prices in this alternative economy, with the source of the multiple equilibria being the endogeneity of asset accumulation and interest rates.[7]

The identification of this equilibrium multiplicity in models with incomplete markets and default is new in the literature. Other mechanisms can be found in the literature as examined in the literature review above. To summarize, they either arise because of the refinancing risk (Calvo 2018, Cole and Kehoe 2000, Lorenzoni and Werning 2013, or more recently He, Krishnamurty, and Milbradt 2018), or because of some strategic complementarity between a small number of countries and risk adverse lenders (Arellano, Bai, and Lizarazo 2018). None of these mechanisms are present in our model, which relies on the endogeneity of the interest rate on the safe asset.

Our main proposition in this section is the ability to rank the multiple world financial-market equilibria according to their welfare properties.

Proposition 2 (Ranking of multiple equilibria) *Multiple equilibria can be ranked according to their aggregate welfare. The higher the number D of consecutive periods before default,*

[7] Auclert and Rognlie (2016) show that the equilibrium is unique in this type of incomplete-insurance market model when prices are exogenously set.

the higher the aggregate welfare will be. As a consequence, the competitive equilibrium that maximizes aggregate welfare is that with the largest admissible D.

The proof can be found in the Appendix. Intuitively, the economy with a lower default amount (and a higher D) is characterized by a higher quantity of liquidity and therefore more risk sharing, which is welfare improving.

4 Identifying distortions in the market economy

We now investigate the distortions in the market equilibrium corresponding to the highest welfare and the largest D. We prove that this equilibrium is not constrained-efficient. Although this equilibrium is the market equilibrium featuring the highest quantity of liquidity, the liquidity remains insufficient for constrained-efficiency, because countries do not completely internalize the social cost of default.

To analyze these distortions more formally, we solve for the allocation of a quasi-planner facing the same risk structure as in the market economy. The quasi-planner can choose how much each country can save, consume, and work but it cannot transfer resources across countries and cannot prevent countries from defaulting.[8] The quasi-planner decides the saving amount of all countries, while internalizing the financial market clearing condition. The individual budget constraint of each country must be fulfilled. In addition, since the quasi-planner cannot forbid default, participation constraints of countries must also hold. The optimal allocation of the quasi-planner defines the constrained-efficient allocation when the pecuniary externality is internalized. Our approach is thus similar to that of Davila, Hong, Krusell, and Rios-Rull (2012), who define a notion of constrained-efficiency in an incomplete insurance-market environment without default. We extend this here to an economy with default, imposing an additional participation constraint on the planner.

We now turn to the formal program of the quasi-planner. We use the same notation as in the market economy of Section 2, except that we add a tilde for the quasi-planner allocation.[9]

[8]The term quasi-planner refers to a planner maximizing social welfare with country-specific budget constraints as in Veracierto (2008) in a different environment, for instance. A general theory for the construction of a quasi-planner economy to identify distortions in an incomplete-insurance market environment is presented in LeGrand and Ragot (2017). We apply it here to economies with default.

[9]To simplify the exposition, we directly assume that productive countries consume the same amount, which is a direct outcome of the labor choice made in those countries by the quasi-planner.

First, the planner chooses the optimal default decision $\tilde{D}$. The quasi-planner then chooses the consumption $\tilde{c}_0$, net savings and $\tilde{B}_0$, and labor supply $\tilde{l}_{k0}$ of productive countries that have been unproductive for $k = -1, 0, \ldots, \tilde{D}$ consecutive periods. The planner also chooses the consumption $\tilde{c}_k$ and the savings $\tilde{B}_k$ of unproductive countries for $k = 1, 2, \ldots, \tilde{D}$ consecutive periods. As before, the net wealth of countries reentering the financial markets is zero, $\tilde{B}_{-1} = 0$. The quasi-planner is concerned with the instantaneous aggregate utility $\tilde{U}$ expressed as:

$$\tilde{U} = \sum_{k=-1}^{\tilde{D}-1} \tilde{n}_{k0} \left(u\left(\tilde{c}_0\right) - \tilde{l}_{k0} \right) + \sum_{k=1}^{\tilde{D}-1} \tilde{n}_k \left(u\left(\tilde{c}_k\right) - \bar{l} \right) \tag{24}$$
$$+ \tilde{n}_p^d \left(u\left(\tilde{c}_p^d\right) - \tilde{l}_p^d \right) + \tilde{n}_{u,1}^d \left(u\left(\tilde{c}_{u,1}^d\right) - \bar{l} \right) + \left(\tilde{n}_u^d - \tilde{n}_{u,1}^d \right) \left(u\left(\tilde{c}_u^d\right) - \bar{l} \right),$$

where the tilde in population shares highlights their dependence in $\tilde{D}$. In equation (24), we distinguish unemployed countries that have just defaulted and reimburse $\underline{B}$. Formal expressions of population shares have the same expression as in the market economy, given in Appendix A. The quasi-planner's program can be expressed recursively as follows:

$$\tilde{W} = \max_{\substack{\{\tilde{q}, \tilde{D}, \tilde{c}_p^d, \tilde{l}_p^d, \tilde{c}_u^d\}, \\ \{\tilde{l}_{k0}\}_{-1 \le k < \tilde{D}-1}, \{\tilde{c}_k\}_{1 \le k < \tilde{D}-1}, \{\tilde{B}_k\}_{0 \le k < \tilde{D}-1}}} \tilde{U} + \beta \tilde{W}', \tag{25}$$

subject to the set of country budget constraints, which are similar to (14)–(16) in the market economy:

$$\tilde{B}_{k-1} = \tilde{c}_k - \bar{l} + \tilde{q}\tilde{B}_k, \quad 1 \le k < \tilde{D} - 1, \tag{26}$$

$$\tilde{B}_{\tilde{D}-2} = \tilde{c}_{D-1} - \bar{l} + \tilde{q}\left(1 - \rho\right)\tilde{B}_{\tilde{D}-1} + \tilde{q}\rho\underline{B}, \tag{27}$$

$$\tilde{c}_0 + \tilde{q}\tilde{B}_0 = \tilde{l}_{k0} + \tilde{B}_k, \quad -1 \le k \le \tilde{D} - 1, \tag{28}$$

$$\tilde{c}_p^d = \varphi \tilde{l}_p^d, \; \tilde{c}_{u,1}^d = \varphi \bar{l} + \underline{B}, \; \tilde{c}_u^d = \varphi \bar{l}, \tag{29}$$

and subject to individual participation constraints (similar to (17) and (19) in the market economy):

$$\tilde{V}_{k0} \ge \tilde{V}_p^d\left(\underline{B}\right), \quad -1 \le k < D, \tag{30}$$

$$\tilde{V}_k \ge \tilde{V}_u^d\left(\underline{B}\right), \quad 1 \le k < D. \tag{31}$$

The quantities $\tilde{V}_{k0}$ and $\tilde{V}_k$ represent the intertemporal welfare of different country types, with expressions similar to (21) and (22) for the market economy. The last constraint of the quasi-planner program is the financial market clearing:

$$\sum_{k=0}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k + (1-\rho)\tilde{n}_{\tilde{D}-1} \tilde{B}_{\tilde{D}-1} + \rho \tilde{n}_{\tilde{D}-1} \underline{B} = 0. \tag{32}$$

The key difference between the market economy and the constrained-efficient allocation is that the quasi-planner internalizes the effect of saving and default decisions on the price of the safe asset, and thus on aggregate risk sharing.

The solution of the quasi-planner program is straightforward. We now present the corresponding first-order conditions in order to provide insights into the constrained-efficient allocation. We denote by λ the Lagrange multiplier of the financial market clearing constraint (32), which corresponds to the social value for the planner of one additional unit of the world safe asset. Euler equations can be written as:

$$\tilde{q} + \lambda = \beta \left(\alpha + (1-\alpha)\, u'\left(\tilde{c}_1\right)\right), \tag{33}$$

$$\tilde{q} u'\left(\tilde{c}_k\right) + \lambda = \beta \left(1 - \rho + \rho u'\left(\tilde{c}_{k+1}\right)\right), \quad 1 \leq k \leq \tilde{D} - 2, \tag{34}$$

$$q\tilde{u}'\left(c_{\tilde{D}-1}\right) + \lambda \geq \beta. \tag{35}$$

The price is determined by the first-order condition:

$$\sum_{k=-1}^{D-1} n_k^p B_k u'\left(c_k^p\right) + \sum_{k=1}^{D-2} n_k^u B_k^u u'\left(c_k^u\right) + \left((1-\rho) n_{D-1}^u B_{D-1}^u + \rho n_{D-1}^u \underline{B}\right) u'\left(c_{D-1}^u\right) = 0. \tag{36}$$

Before commenting on these expressions, it is worth noting that when $\lambda = 0$, i.e. when there is no internalization of the effect of saving on prices, we find the same first-order conditions as in the market economy. The difference between the two allocations therefore stems solely from the internalization of the pecuniary externality.

The internalization of the pecuniary externality affects the Euler equations through the additional net gain of saving, which is captured by λ. Furthermore, the quasi-planner's price setting implies an additional first-order condition, which is equation (36). This equation stipulates that the quasi-planner sets the price of the safe asset such that the redistributive effects maximize welfare across buyers and sellers. What does the quasi-planner's allocation look like? Before presenting the main result, it is worth noting that the quasi-planner solves a well-known insurance problem in an incomplete insurance market economy. In an economy with exogenous borrowing constraints (and no default) where the planner can supply a store of value at no cost, Bewley (1983) showed that the optimal allocation is the full-insurance allocation, but that this allocation cannot be exactly reached because the quantity of liquidity would be infinite. In an economy with default, a full-insurance allocation can be exactly reached, but only by countries participating in the world financial markets.

To see this more formally, observe that a solution to equations (33)–(35) is $\tilde{q} + \lambda = \beta$ and $u'(\tilde{c}_k) = 1$ for $k = 0, \ldots, \tilde{D} - 1$. The quasi-planner distorts countries' saving choices so as to reach full risk sharing for countries participating in the financial markets. In this case, the welfare maximization constraint (36) is equivalent to the market equilibrium (32). Compared to Bewley (1983), full risk sharing can be reached because it concerns only a subset of countries. It fosters the intertemporal welfare of participating countries, and therefore reduces the default incentives. This maximizes the safe asset quantity and aggregate welfare. The next proposition summarizes this quasi-planner allocation. The formal proof is provided in Appendix D.

Proposition 3 (Constrained-efficient allocation) *The constrained-efficient allocation is either autarky or is characterized by $\tilde{c}_k = u'^{-1}(1)$ for $k = 0, \ldots, \tilde{D} - 1$.*

First, one cannot exclude the possibility that autarky might be the only equilibrium, following the results of Bulow and Rogoff (1989). It is the case if the cost of default is low, for instance $\varphi = 1$. We now focus on the case where the cost of default is high enough for borrowing to be sustained in equilibrium (which is the case in the numerical example below). In this case, the optimal outcome is always to implement full risk sharing among participating agents. As a direct consequence, the constrained-efficient allocation differs in the general case from the market allocation characterized in Section 3.1. To see this, observe that $\tilde{c}_k = u'^{-1}(1)$ in the market economy implies $q = \beta$. However, in this case, there is no reason for the saving decisions B_k, $k \leq \tilde{D} - 1$ to satisfy the financial market clearing condition. Since the quasi-planner internalizes the financial market clearing condition, it distorts the saving decision to obtain full risk sharing and equilibrium in the financial markets.

This remark provides an intuition for the device presented in the next section. Perfect risk sharing implies an increase in the liquidity available for self-insurance in the market economy, but without distorting individual Euler equations. Again, the intuition for the optimal provision of safe assets is the same as the monetary setting of Bewley (1983).

5 A welfare-improving device: An international fund

Based on the insight of the previous analysis, we now show that an international fund can improve welfare. We assume that the fund has no comparative advantage compared to the financial markets. In particular, the fund cannot transfer resources according to countries' production status (otherwise the fund could implement the first best allocation by completing the market).[10]

In addition, we make the following assumptions:

1. The fund can issue debt and is financed solely by the contributions of member countries.

2. The fund cannot implement transfers that are conditional on the productive status of member countries.

3. Countries can freely choose to belong to the fund and can always choose to opt out.

4. Contributing countries agree not to transact with countries that do not contribute to the fund.

First, the fund budget is only financed by the contribution of countries. Second, and as explained above, we rule out contributions that are contingent on the productive status. Third, countries can decide whether to participate in the fund or not. The participation decision is sovereign. Last, contributing countries agree, when joining the fund, not to transact with non-contributing countries. Otherwise, countries would have no incentive to pay their contribution to the fund, as they could benefit from world liquidity (and pay the world interest rate on any safe asset through the absence of arbitrage) without paying any contributory costs.

We now turn to the formalization of the fund. The environment is similar to the market economy described in Section 2 (we keep the same notation), except for the additional presence of the fund, which issues one-period, risk-free debt financed by the contributions of member states. We denote by F' the debt issued by the fund at price q in a given period, while F is the amount the fund repays. Note that the fund pays the world risk-free interest rate, because there

[10] One could argue that the ability to lend to countries in crisis against some credible commitment from these countries (a program) is a comparative advantage for certain international institutions, such as the IMF. This comparative lending advantage helps to complete the market. We do not consider this role here, but instead focus on the supply of liquidity.

is no arbitrage. The measure of countries participating in the fund is denoted by m_p. Countries participating in the fund pay τ. Using this notation, the fund budget constraint is:

$$F = qF' + m_p \tau. \tag{37}$$

The world financial market clearing condition can now be expressed as:

$$\sum_{k=0}^{D-2} n_k B_k + (1-\rho) n_{D-1} B_{D-1} + \rho n_{D-1} \underline{B} = F'. \tag{38}$$

Finally, the budget constraints of countries are the same as (14)–(16), except that countries additionally contribute to the fund by an amount τ.

Full-risk sharing can be implemented for participating countries, as shown by the next Proposition.

Proposition 4 (Equilibria in family-head and decentralized economies) *For finite D, there exists a value of F such that full-risk sharing is implemented in steady-state among countries participating to the fund.*

The proof can be found in Section E of the Appendix. The equilibrium quantity F is determined to guarantee that the asset price is $q = \beta$. In this equilibrium all countries participating in the financial markets contribute to the fund. The proposition remains silent regarding existence. Indeed, existence is not guaranteed in the general case. Instead, we provide below a numerical example to show existence and the properties of such a fund in a quantitatively relevant environment. The fund can be seen as a pure financial actor targeting the proper interest rate using the size of its balance sheet in this non-Ricardian environment. This fund is obviously reminiscent of the IMF issuing Special Drawing Rights, which are international stores of value. SDRs are reserve assets and have been issued by the IMF since 1970, with the explicit goal of reducing the world liquidity shortage (see Williamson 2009 for a summary of the history of the introduction of SDRs). A first difference between the fund introduced in this section and SDRs is that the interest rate on assets issued by the fund is a yearly interest rate, whereas the remuneration of SDRs is an average of short-term (3 month) interest rates on a basket of currencies.[11] The next section quantifies the optimal amount of this world liquidity.

[11] The weekly interest rates on SDRs are provided at `http://www.imf.org/external/np/fin/data/sdr_ir.aspx`, together with the explicit interest rate formula.

6 A numerical example

To provide further insights, we now perform a quantitative exercise to investigate equilibrium multiplicity and to quantify the size of the fund required to maximize aggregate welfare. To calibrate the model we use the literature on both sovereign default and on disaster events, which is consistent with the two-state Markov structure we use for the idiosyncratic risk. The period is one year. The model has eight parameters, for which eight targets or values are provided.

The curvature of the utility function is set to $\sigma = 2$, as in Arellano (2008), and the discount factor is set to $\beta = 0.96$ to obtain an annual world interest rate of around 4% in the world-market economy.

We set the probability of remaining productive to 96.5% such that the probability of moving from a productive to unproductive state is 3.5%. This is equal to the annual probability of entering into a disaster state, as found by Barro and Ursua (2012). The probability of remaining unproductive is set to 0.87% such that the average number of periods spent in a unproductive state amounts to 7.7 years. This value is consistent with the average duration of a disaster state. The production when unproductive is set to $\bar{l} = 0.99$, which implies a cumulative output loss before default of 7%. This is consistent with Tomz and Wright (2013), who report that output is 8% below trend when default occurs. The probability of reentering the economy is set to $\theta = 15\%$, which generates an average length of financial market exclusion of 7 years, which matches the length reported by Tomz and Wright (2013). Finally the cost of default is set to 0.6% of output ($\varphi = 0.994$). This value is chosen to trigger default after 7 consecutive periods in the unproductive state (one possible equilibrium, see below), which is the average length of a disaster. This value implies that 16.4% of all countries are in default, which is close to the empirical value of 19% found by Tomz and Wright (2013).

Table 1 summarizes the parameter and the targeted values, with the references that we use. An additional untargeted outcome of this calibration is that the unconditional probability of default amounts to 1.0%. This value is slightly below empirical estimates, which vary between 1.8% and 2.2%. We now investigate the equilibrium properties and the optimal size of the fund.

The first outcome of the exercise is that multiple equilibria are pervasive. The previous calibration is consistent with an equilibrium where $D = 7$, i.e. where default occurs after 7 years in an unproductive state, but is also consistent with a range of equilibria, where D

Parameters	Values	Target and references and references
Discount factor	$\beta = 0.96$	Interest rate of 4%
Utility function	$\sigma = 2$	Arellano (2008)
Persistence of good state	$\alpha = 96.5\%$	3.5% disaster probability (Barro and Ursua 2012)
Persistence of disaster	$\rho = 87\%$	Average duration of disaster of 7.7 years (Barro and Ursua 2012)
Labor in disaster state	$\bar{l} = 0.99$	7% cumulative GDP fall on default (Tomz and Wright 2013)
Prob. of reentering	$\theta = 15\%$	7 years of exclusion after default (Tomz and Wright 2013)
Default cost	$\varphi = 0.994$	Default after 7 years in unproductive state (Barro and Ursua 2012)
Debt recovery	$\underline{B} = -0.05$	Haircut of 40% (Tomz and Wright 2013)

Table 1: Calibration

varies up to 13. As was explained in the previous section, the source of equilibrium multiplicity is the endogeneity of the benefit of participating in financial markets, which depends on the default decision. For instance, we find equilibria for $D = 7$ to $D = 13$. In Table 2, we report certain key statistics for all equilibria: interest rate, the fraction of countries in default, and the unconditional default probability.

D	7	8	9	10	11	12	13
Interest rate (%)	4.16	4.155	4.15	4.15	4.15	4.15	4.15
Share of countries in default (%)	16.4	14.4	12.3	11.0	9.7	8.5	7.4
Unconditional default probability (%)	1.0	0.8	0.7	0.65	0.6	0.5	0.4
Welfare (level)	-2.36	-2.07	-1.82	-1.60	1.41	-1.23	-1.08
Haircut (%)	40.1	40.5	40.9	41.1	41.4	41.6	41.8

Table 2: Equilibrium properties for various equilibria indexed by D

The interest rate decreases slightly when D increases, from 4.16% when $D = 7$ to 4.15%

when $D = 13$. When D rises, both the supply and the demand for liquidity rise, as a smaller number of countries default and countries need to self-insure for a higher number of periods in an unproductive state before defaulting. Consequently, the interest rate does not change significantly when D rises. In addition, and as can be expected, both the fraction of countries in default and the equilibrium default probability decrease with D, and are too low compared to the data. The welfare comparisons across these equilibria are presented below.

As shown in Section 3.3, more risk sharing takes place in equilibria with larger D. We illustrate this aspect in Figure 1, where we plot the consumption profiles as a function of the number of consecutive unproductive periods for different values of D. In the interests of clarity, we only report the profiles for $D = 7$, 10, and 13. The path of consumption decreases with the

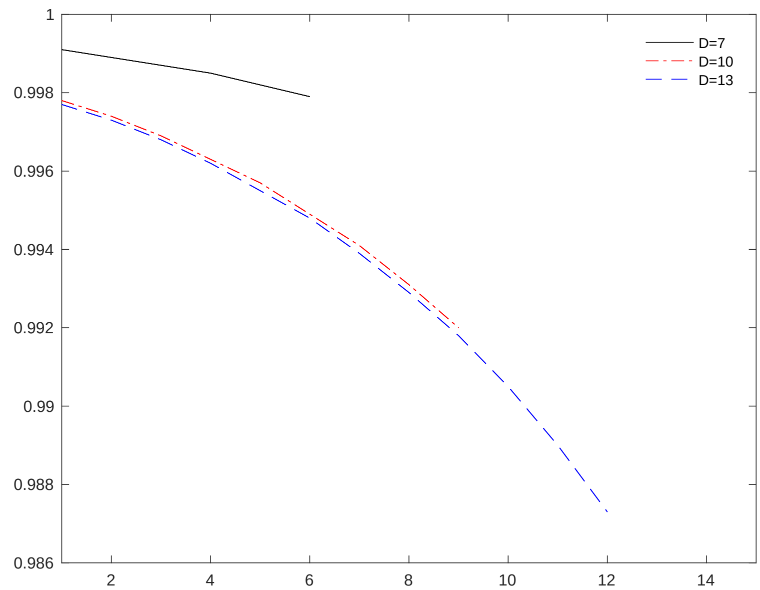

Figure 1: Consumption profiles of participating countries as a function of the number of consecutive unproductive periods, for different equilibria, indexed by D.

number of consecutive periods in an unproductive state (before default). For a given number of consecutive periods in the non-productive state (say 4 for instance), the consumption is decreasing with D. As a consequence, more risk-sharing in the extensive margin (i.e., higher D) is accompanied by less risk-sharing in the intensive margin (i.e., for the same number of period in the unproductive states) among these equilibria.

We now compute the optimal size of the fund that maximizes aggregate welfare. Iterating over the size of the fund F, we find that the welfare-maximizing value $D^{opt} = 16$: countries default after being unproductive for 16 periods. The world interest rate is 4.17%, which is the complete market interest rate $1/\beta - 1$. All countries participating in financial markets, either productive or not, enjoy the same consumption, equal to 1. There is thus full risk sharing for countries participating in the financial markets.

This value of D is attained for a size of the fund equal to 13.2% of world GDP. The average contributions of a country to the fund is equal to less than 0.6% of GDP ($\tau = 0.55\%$).[12] In this economy, the fraction of countries defaulting is 0.3% and only 5.0% of the countries are excluded from financial markets.

We now compare the welfare properties of the different equilibria. For every equilibrium, we compute the aggregate welfare using the standard utilitarian criterion (see equation 23), which equally weights the intertemporal welfare of all countries. In Table 3, we report the aggregate welfare differences between market equilibria (for D varying from 7 to 13) and the fund economy where $D^{opt} = 16$. This aggregate welfare difference is expressed as a consumption equivalent and corresponds to the relative fall in consumption in each country necessary for the welfare in the fund economy to be identical to that in the market economy for a given D.

D	7	8	9	10	11	12	13
Welfare difference (as consumption equivalent %)	0.06	0.05	0.04	0.03	0.03	0.02	0.01

Table 3: Welfare difference between market equilibria and the fund economy expressed as consumption equivalent (in percentages differences).

When $D = 7$, we find that the welfare gain of the fund introduction represents a consumption increase in all countries of 0.06% compared to the consumption in the market economy. This welfare gain decreases with D. The welfare gain provided by the fund introduction comes from two sources, an intensive and an extensive margin. First, countries are better insured in the fund economy when they participate in financial markets, which is the intensive margin. Second, less countries default in the fund economy as D is higher, which is the extensive margin. We find that most of the welfare gain (90%) comes from the extensive margin, and the associated reduction in default probabilities. As a consequence, one should think about the fund as a liquidity device that enables to lower the number of defaults at the equilibrium.

[12] Although relatively small, the fund is nonetheless much larger than the current amount of outstanding IMF SDRs, which amounted to less than 0.3% of world GDP in 2016.

7 An asymmetric world: The Hegemon

The analysis above focused on the case where every country was a price-taker and had no market power. We now check that our qualitative results are robust to the introduction of a country having some market power in world financial markets. Indeed, there is no consensus on the "best" model for the world financial market (see Eichengreen 2011, for instance). We follow Farhi and Maggiori (2018) and introduce a country called the Hegemon into the previous economy. This country has a positive mass and internalizes the impact of its choices on the asset price. More precisely, we consider the case where a country is larger than other countries and wields market power on the world financial market. This extension aims to show that the main results of our analysis are robust to the relaxation of the assumption of perfectly symmetric countries. In particular, even in this environment, excess default remains at the equilibrium and an international liquidity provider can restore constrained-efficiency. We now assume that the world includes, in addition to the continuum of countries presented above, a country referred to as the Hegemon, of mass m with financial market power. The Hegemon has the same preferences as any other country. Clearing on the world financial market can now be expressed as follows:

$$mB^H + \int_{i\in I} q\left(B'^i, s^i\right) B'^i G\left(di\right) = 0, \tag{39}$$

where B^H denotes the net borrowing demand of the Hegemon. Other notation remains unchanged.

As the Hegemon has a positive mass, it internalizes the effect of its (net) borrowing demand B^H on the price of the world safe asset $q\left(B^H\right)$:

$$V^H\left(B^H, X\right) = \max_{\{c^H, B^{H\prime}\}} u\left(c^H\right) + \beta V^H\left(B^{H\prime}, X'\right), \tag{40}$$

$$\text{s.t. } c^H = y^H + B^H - q\left(B^{H\prime}\right) B^{H\prime}. \tag{41}$$

In the interests of conciseness, we derive analytical properties in this new environment in Section G of the Appendix. First, we prove that the constrained-efficient allocation is characterized by perfect insurance and an asset price equal to β. This characterization is similar to that of Proposition 3 in the absence of the Hegemon. Furthermore, we determine the net position of the Hegemon as a function of its revenues. If revenues are large, the Hegemon will be a net borrower, with the reverse also holding.

Our second result is that the market equilibrium does not generate the constrained-efficient allocation. The asset price at the market equilibrium is higher and the equilibrium features imperfect insurance. Defaults at the equilibrium are too high because there are insufficient safe assets in the economy. In other words, the result found for the symmetric equilibrium, where there is excess default at the equilibrium, still holds in the presence of a Hegemon with financial market power. As in the symmetric economy, a world fund providing liquidity to all countries can increase the aggregate welfare. The fund turns out to be a net liquidity provider, which induces a fall in the price of liquidity and neutralizes the market power of the Hegemon.

8 Conclusion

We provide a tractable model where sovereign default can be studied in general equilibrium. This model allows us to derive positive and normative properties of the equilibrium structure implied by the endogenous quantity of the world safe asset. A salient property of the model is equilibrium multiplicity, which results from an endogenous world interest rate. In addition, all market equilibria exhibit insufficient risk sharing compared to a constrained-efficient equilibrium. The additional result of the paper is that welfare is increased by introducing a fund that issues safe assets based on the voluntary contributions of member states. The liability of the fund is around 13% of world GDP and in the benchmark case no country has market power. The fund is found to mainly contribute to decrease the number of equilibrium defaults. The fund size is much larger than the outstanding SDRs issued by the IMF. We generalize the setting to the case where a country is large enough to wield some market power on the world financial markets. The tractability of the framework allows for different extensions. For instance, one could introduce different debt maturities to investigate the properties of equilibrium portfolio. We leave this for future research.

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Appendix

A Population shares

Using the model setup presented in Section 2.1 and the equilibrium presented in Section 3.1, we deduce the following population shares:

$$n_0 = \frac{\theta}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\rho}{2-\rho-\alpha}, \tag{42}$$

$$n_k = \rho^{k-1}(1-\alpha)n_0 \text{ for } k = 1, \ldots, D-1, \tag{43}$$

$$n_k = 0 \text{ for } k \geq D. \tag{44}$$

We denote by n_u^d and n_p^d the mass of unproductive and productive agents who have defaulted and are excluded from financial markets and by $n_{u,1}^d$ the mass of countries who have just defaulted. We have the following expressions:

$$n_p^d = \frac{(1-\theta)(1-\alpha)\rho^{D-1}}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\rho}{2-\rho-\alpha}, \tag{45}$$

$$n_{u,1}^d = \rho^D(1-\alpha)n_0, \tag{46}$$

$$n_u^d = \frac{(1-(1-\theta)\alpha)\rho^{D-1}}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\alpha}{2-\rho-\alpha}. \tag{47}$$

With our timing convention, the number of excluded countries re-entering financial markets after having being excluded is then $n_{-10} = \theta \left(\alpha n_p^d + (1-\rho)\, n_u^d\right)$ in each period. It is then straightforward to deduce that the overall population (excluded and participating agents) of productive countries amounts to $\frac{1-\rho}{2-\alpha-\rho}$ and this of unproductive countries to $\frac{1-\alpha}{2-\alpha-\rho}$. Note that these populations are independent of the quantity D.

B Proof of Proposition 1

The proof can be split into three steps.

B.1 Proof that: $q \geq \beta$.

Let us assume $q < \beta$. First, if $D < \infty$, equation (13), implies $u'(c_{D-1}) \geq \beta/q > 1$ and $u'(c_k) > 1$ for $k = 1, \ldots, D-1$ by induction from equation (12). Equation (11) then yields $q > \beta$, which is a contradiction. Second, if $D = \infty$, we first show from (11) and (12) that $1 > u'(c_k)$ for all k. Equation (12) also implies $\rho(u'(c_{k+1}) - u'(c_k)) < (1-\rho)(u'(c_k) - 1) < 0$ and $(u'(c_{k+1}))_k$ is decreasing and positive. It thus converges to u'_∞, with $u'_\infty \left(1 - \frac{\beta}{q}\right) = \frac{\beta}{q}(1-\rho)(u'_\infty - 1)$ using (12). Since $q < \beta$, we must have $u'_\infty > 1$, which contradicts $1 > u'(c_k)$ for all k.

B.2 Case $D = \infty$

Assume that $q = \beta$. Using the Euler conditions (11) and (12), we recursively show that $u'(c_k) = 1$ for all k. Budget constraint then implies $u'^{-1}(1) + \beta B_{k+1} = \bar{l} + B_k$. It is then easy to show that $\lim_{k\to\infty} B_k = -\infty$ (any other case being impossible due to financial market clearing condition). This is a contradiction with equation (9) as the there is a threshold for debt in any equilibrium under consideration. In consequence $q > \beta$. We thus obtain from Euler equations (11)–(13):

$$1 - \alpha < (1-\alpha)u'(c_1)$$

$$u'(c_k) - 1 < \rho\left(u'(c_{k+1}) - 1\right), \quad k \geq 1,$$

which implies $u'(c_k) - 1 \geq \rho^{-k}(u'(c_1) - 1)$ with $u'(c_1) > 1$. In consequence, $\lim_{k\to\infty} u'(c_k) = \infty$ and $\lim_{k\to\infty} c_k = 0$. Budget constraint (14) implies $\lim_{k\to\infty} B_k = -\frac{\bar{l}}{1-q}$.

B.3 Case $D < \infty$

Since the default occurs at date D, we have $V_k \geq V_u^d(\underline{B})$ for $k = 1, \ldots, D-1$ and $V_{k0} \geq V_p^d(\underline{B})$ for $k = -1, 0, \ldots, D-1$. Let assume that $V_{D-1,0} > V_p^d(\underline{B})$. We have from equations (15) and (22):

$$V_{D-1} = u\left(B_{D-2} + \bar{l} - qB_{D-1}\right) - \bar{l} + \beta\left((1-\rho)V_{D-1,0} + \rho V_u^d(\underline{B})\right),$$

with $V_{D-1} \geq V_u^d(\underline{B})$ and V_{D-1} decreasing in B_{D-1}. Therefore, the agent u unproductive for $D-1$ periods could increase her welfare by decreasing slightly B_{D-1}. This would not affect default incentives since $V_{D-1,0} > V_p^d(\underline{B})$, which by continuity still holds after a small decrease in B_{D-1}. This contradicts the optimality of agent's choices and this implies $V_{D-1,0} = V_p^d(\underline{B})$.

C Proof of Propositions 2

Let $D \geq 1$. Using equations (42)–(46) of Section A in Appendix, we have:

- n_0 goes up with D:
$$\frac{\partial n^p}{\partial \rho^{D-1}} = -\frac{(1-\theta)(1-\alpha)n^p}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} < 0.$$

- n_p^d goes down with D since the total population of p agents remains constant.

- $\sum_{k=1}^{D-1} n_k$ goes up with D:
$$\frac{\partial n^u}{\partial \rho^{D-1}} = -\frac{(1-\theta)(1-\alpha)n^p}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} - \frac{\theta}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\alpha}{2-\alpha-\rho} < 0.$$

- n_u^d goes down with D since the total population of u agents remains constant.

Consider the planner program that can rewritten as:

$$W = \max_{D \in \mathbb{N}_*} \max_{\substack{\{c_p^{d,D}, l_p^{d,D}, c_u^{d,D}\}, \\ \{l_{k0}^D\}_{-1 \le k < D}, \{c_k^D, B_k^D\}_{0 \le k < D} \in \mathcal{A}_D}} U + \beta W',$$

where $\mathcal{A}_D$ is the feasible set for a given $D \in \mathbb{N}_*$. We have added superscripts D to allocations in order to highlight the dependence in D. We have:

$$\begin{aligned} U = & \sum_{k=-1}^{D-1} n_{k0}^D \left(u\left(c_0^D\right) - l_{k0}^D \right) + \sum_{k=1}^{D-1} n_k^D \left(u\left(c_k^D\right) - \bar{l} \right) \\ & + n_p^{d,D} \left(u\left(c_p^{d,D}\right) - l_p^d \right) + n_{u,1}^{d,D} \left(u\left(c_{u,1}^{d,D}\right) - \bar{l} \right) + \left(n_u^{d,D} - n_{u,1}^{d,D} \right) \left(u\left(c_u^{d,D}\right) - \bar{l} \right), \end{aligned}$$

where we also added the D superscript to population proportions. Let us show that $\mathcal{A}_D \subset \mathcal{A}_{D'}$ for any $D' \ge D$. To do so, we consider a feasible allocation that can be characterized by $(c_p^{d,D}, l_p^{d,D}, c_u^{d,D}, \{l_k^D\}_{-1 \le k < D}, \{c_{p,k}^D, B_k^D\}_{0 \le k < D}) \in \mathcal{A}_D$. Using budget constraint (1), we have:

$$B_{k-1}^{u,D} \ge c_k^{u,D} - \bar{l} + pB_k^{u,D}, 0 \le k < D-1, \tag{48}$$
$$B_{D-2}^{u,D} \ge c_{D-1}^{u,D} - \bar{l} + p\left(1-\rho\right) B_{D-1}^{u,D}, \tag{49}$$
$$c_{u,1}^{d,D} \le \varphi \bar{l} + \underline{B}, \tag{50}$$
$$c_u^{d,D} \le \varphi \bar{l}, \tag{51}$$
$$c_{p,k}^{p,D} + pB_0^{p,D} \le l_k^{p,D} + B_k^{p,D}, 0 \le k < D, \tag{52}$$
$$c_p^{d,D} \le \varphi l_p^{d,D}. \tag{53}$$

From the previous remark about the impact of D on population shares, we know that the population of p and u agents who are excluded is shrinking, while the population of p and u

countries who participate is going up. We define for unproductive countries:

$$B_k^{D'} = \begin{cases} B_k^D & \text{for } n_k^D \text{ countries} \\ 0 & \text{for } n_k^{D'} - n_k^D \text{ countries} \end{cases},$$

$$c_k^{D'} = \begin{cases} c_{p,k}^D & \text{for } n_k^D \text{ countries} \\ c_{p,k}^d & \text{for } n_k^{D'} - n_k^D \text{ countries} \end{cases},$$

$$l_k^{D'} = \begin{cases} \bar{l} & \text{for } n_k^D \text{ countries} \\ \varphi\bar{l} & \text{for } n_k^{D'} - n_k^D \text{ countries} \end{cases},$$

$$\begin{aligned} c_u^{d,D'} &= c_u^{d,D} \text{ for } n_u^{d,D'} - n_{u,1}^{d,D'} \text{ countries,} \\ c_{u,1}^{d,D'} &= c_{u,1}^{d,D} \text{ for } n_{u,1}^{d,D'} \text{ countries,} \\ l_u^{d,D'} &= \varphi\bar{l} \text{ for } n_u^{d,D'} \text{ countries,} \end{aligned}$$

where the idea is to "view" some excluded agents for D as participating agents for D', which is possible since we have (i) $n_k^D \le n_k^{D'}$, (ii) $n_u^{d,D'} \le n_u^{d,D}$, and (iii) $n_k^D + n_u^{d,D} = n_k^{D'} + n_u^{d,D'}$. It is obvious that constraint (50) holds for D', and that (48) and (49) also hold for D' (for a population of n_k^D agents that were in the same state for D). For the remaining population of $n_k^{D'} - n_k^D$ agents, equation (48) for D' becomes $c_u^{d,D'} - \bar{l} = c_u^{d,D} - \bar{l} + 0 = (\varphi - 1)\bar{l} \le 0$, which therefore holds. By the same token, equation (49) for D' also holds.

For productive agents, we similarly define:

$$B_0^{D'} = \begin{cases} B_0^D & \text{for } n_0^D \text{ countries} \\ 0 & \text{for } n_0^{D'} - n_0^D \text{ countries} \end{cases},$$

$$c_0^{D'} = \begin{cases} c_0^D & \text{for } n_0^D \text{ countries} \\ c_p^{d,D} & \text{for } n_0^{D'} - n_0^D \text{ countries} \end{cases},$$

$$l_{k0}^{D'} = \begin{cases} l_{k0}^{p,D} & \text{for } n_{0k}^D \text{ countries} \\ \varphi l_p^{d,D} & \text{for } n_{0k}^{D'} - n_{0k}^D \text{ countries} \end{cases},$$

$$\begin{aligned} c_p^{d,D'} &= c_p^{d,D} \text{ for } n_p^{d,D'} \text{ countries,} \\ l_p^{d,D'} &= \varphi l_p^{d,D} \text{ for } n_p^{d,D'} \text{ countries,} \end{aligned}$$

Note that equation (53) holds for D' by construction. Regarding, equation (52), we have $c_p^{d,D} \leq \varphi l_p^{d,D} \leq l_p^{d,D} \leq l_p^{d,D'}$, which therefore holds.

In consequence the allocation $\left(c_p^{d,D}, l_p^{d,D}, c_{u,1}^{d,D}, c_u^{d,D}, \{l_k^D\}_{-1 \leq k < D}, \{c_k^D, B_k^D\}_{0 \leq k < D}\right)$ belongs to the feasible set $\mathcal{A}_{D'}$. We deduce that $\mathcal{A}_D \subset \mathcal{A}_{D'}$. This has two consequences.

1. For a given price, feasible sets are increasing in D (in the sense of inclusion). This implies that any welfare level for D can be reached for any $D' \geq D$. The welfare is therefore increasing in D.

2. By the same token, we can therefore rank competitive equilibria by an aggregate welfare criterion using D. This proves Proposition 2.

D Proof of Proposition 3

We show that the quasi planner allocation is either autarky or it is characterized by is $\tilde{q} + \lambda = \beta$ and $u'(\tilde{c}_k) = 1$ for $k = 0, \ldots, \tilde{D} - 1$.

As a preliminary, note that for some parameters values, the only equilibrium is the no-trade equilibrium characterized by $\tilde{B}_k = 0$ for all k. Following the results of Bulow and Rogoff (1989), it can be the case if φ (the output cost of default) is close to 1. In what follows, we assume that it is not the case and that there is a k such that $B_k > 0$. We proceed in three steps.

Step 1. If $\tilde{q} + \lambda = \beta$, equations (33)–(35) imply $u'(\tilde{c}_1) = 1$, and then $u'(\tilde{c}_k) = 1$ for $k = 1, \ldots, \tilde{D} - 1$. Condition (36) holds (as in any equilibrium).

Step 2. We show that there is no constrained-efficient equilibrium where $\tilde{q} + \lambda > \beta$. We proceed by contradiction. Assume that $\tilde{q} + \lambda > \beta$. First, from equation (33), we obtain $u'(\tilde{c}_k) > 1$. Then using (34), we recursively show that $u'(\tilde{c}_{k+1}) > u'(\tilde{c}_k)$, for $k = 0, \ldots, \tilde{D} - 1$ and $(\tilde{c}_k)_k$ is decreasing. Second, budget constraints (26) then imply that $(B_k)_k$ is decreasing.

We now show that the financial market clearing condition (32) and the first-order condition (36) cannot hold at the same time. Let us define k_0 as the index such that $B_{k_0} \geq 0 > B_{k_0+1}$. The index k_0 exists because $(B_k)_k$ is decreasing and equality (32) holds. This implies $\tilde{B}_k \geq 0$

for all $k = 0, \ldots, k_0$ and $\tilde{B}_k < 0$ for $k = k_0 + 1, \ldots, \widetilde{D} - 2$. Since $(\tilde{c}_k)_k$ is decreasing, we deduce:

$$\sum_{k=0}^{k_0} \tilde{n}_k \tilde{B}_k u'(\tilde{c}_k) < u'(\tilde{c}_{k_0}) \sum_{k=0}^{k_0} \tilde{n}_k \tilde{B}_k, \tag{54}$$

as well as:

$$-\left(\sum_{k=k_0+1}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k u'(\tilde{c}_k) + \Big((1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho \tilde{n}_{\tilde{D}-1}\underline{B}\Big) u'(\tilde{c}_{D-1}) \right) > \tag{55}$$
$$-u'(\tilde{c}_{k_0+1}) \left(\sum_{k=k_0+1}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k + (1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho \tilde{n}_{\tilde{D}-1}\underline{B} \right).$$

Furthermore, the financial market clearing condition (32) implies:

$$\sum_{k=0}^{k_0} \tilde{n}_k \tilde{B}_k = -\left(\sum_{k=k_0+1}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k + (1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho \tilde{n}_{\tilde{D}-1}\underline{B} \right),$$

with which equations (54) and (55) implies:

$$-\left(\sum_{k=k_0+1}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k u'(\tilde{c}_k) + \Big((1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho \tilde{n}_{\tilde{D}-1}\underline{B}\Big) u'(\tilde{c}_{D-1}) \right) > \sum_{k=0}^{k_0} \tilde{n}_k \tilde{B}_k u'(\tilde{c}_k).$$

The last inequality shows that the condition (36) cannot hold, what is a contradiction.

Step 3. We consider the case $\tilde{q} + \lambda < \beta$ and show that no constrained-efficient equilibrium exists. The proof is analogous to the one in the second step, except that $(\tilde{c}_k)_k$ and $(\tilde{B}_k)_k$ are shown to be increasing (instead of increasing). We still obtain a contradiction by showing that the financial clearing (32) and the first-order condition (36) cannot simultaneously hold.

E Proof of Proposition 4

An equilibrium in the fund economy is a default horizon D, a fund policy (F, τ), an asset price q, allocations $c_p^d, l_p^d, c_u^d, \{l_k\}_{-1 \le k < D}, \{c_k, B_k\}_{0 \le k < D}$, such that: 1) for a given q and (F, τ), allocations are consistent with individual country programs; 2) financial market clears and (38) holds; 3) the fund policy is balanced and (37) holds. It is straightforward to show that Proposition 1 holds, as the proof does not depend on contributions.

Then full risk-sharing is implemented in the fund economy when $q = \beta$. Indeed, as and $q = \beta$, the first-order conditions, identical to (11)–(13), for $k = 0, \ldots, D-1$ yield $c_0 = c_k = u'^{-1}(1) \equiv c$.

Using the tractability our our framework, we exhibit the 3 equations determing the size of the fund for any finite D.

First equation. The budget constraints of unproductive countries, now including the payment τ, lead after backward iteration to:

$$B_k = \frac{B_0}{\beta^k} + (c - \bar{l} + \tau)\frac{1-\beta^{-k}}{1-\beta}, \tag{56}$$

where B_0 is the saving of participating productive countries. The welfare of participating unproductive countries is:

$$V_k = u(c) - \bar{l} + \beta(1-\rho)V_p^c(0) + \beta(1-\rho)B_k + \beta\rho V_{k+1},$$

which yields after forward iteration:

$$\begin{aligned} V_1 = {} & \Big(u(c) - \bar{l} + \beta(1-\rho)V_p^c(0)\Big)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho} \\ & + (c - \bar{l} + \tau)\frac{\beta(1-\rho)}{1-\beta}\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho} \\ & + \left(B_0 - \frac{c-\bar{l}+\tau}{1-\beta}\right)(1-\rho^{D-1}) + (\beta\rho)^{D-1}\,V_u^d(\overline{B}). \end{aligned} \tag{57}$$

The welfare of a productive participating country with a wealth 0 in the fund economy is:

$$V_p^c(0) = u(c) - (c + \tau + \beta B_0) + \beta\alpha V_p^c(0) + \beta\alpha B_0 + \beta(1-\alpha)V_1,$$

or using the expression (57) of V_1:

$$\begin{aligned} (1-\beta\alpha)\,V_p^c(0) = {} & u(c) - (c+\tau+\beta B_0) + \beta\alpha B_0 \\ & + \Big(u(c) - \bar{l} + \beta(1-\rho)V_p^c(0)\Big)\beta(1-\alpha)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho} \\ & + (c-\bar{l}+\tau)\frac{\beta^2(1-\alpha)(1-\rho)}{1-\beta}\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho} \\ & + \beta(1-\alpha)\left(B_0 - \frac{c-\bar{l}+\tau}{1-\beta}\right)(1-\rho^{D-1}) + (\beta\rho)^{D-1}\beta(1-\alpha)V_u^d(\overline{B}). \end{aligned} \tag{58}$$

As $V_p^c(B_{D-1}) = V_p^d(\overline{B})$, we have $V_p^c(0) = V_p^d(\underline{B}) - B_{D-1}$ or using (56):

$$V_p^c(0) = V_p^d(\underline{B}) - \beta^{-(D-1)}B_0 - \frac{1-\beta^{-(D-1)}}{1-\beta}(c - \bar{l} + \tau). \tag{59}$$

Substituting (59) into (58), we obtain:

$$\Psi_1 \tau = \Psi_2 - \Psi_3 B_0, \tag{60}$$

with:

$$\begin{aligned}
\Psi_1 &= \beta^2(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\frac{\beta^{-(D-1)}}{1-\beta} \\
&\quad + \frac{1}{1-\beta}\left(\rho^{D-1}\beta(1-\alpha) - \beta^{-(D-1)}(1-\beta\alpha)\right), \\
\Psi_2 &= -\left(u(c)-\bar{l}\right)\left(1+\beta(1-\alpha)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\right) \\
&\quad - (c-\bar{l})\left(\beta^2(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\frac{\beta^{-(D-1)}}{1-\beta}\right. \\
&\qquad \left. + \frac{1}{1-\beta}\left(\rho^{D-1}\beta(1-\alpha) - \beta^{-(D-1)}(1-\beta\alpha)\right)\right) \\
&\quad \left(1-\beta\alpha-\beta^2(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\right)V_p^d(\underline{B}) - (\beta\rho)^{D-1}\beta(1-\alpha)V_u^d(\underline{B}), \\
\Psi_3 &= \left(1-\beta\alpha-\beta(1-\alpha)\frac{\beta(1-\rho)+(1-\beta)(\beta\rho)^{D-1}}{1-\beta\rho}\right)\beta^{-(D-1)}.
\end{aligned}$$

The two value functions $V_p^d(\underline{B})$ and $V_u^d(\underline{B})$ are endogenous values, as the probability to reenter is positive. The functions can be expressed as function of $V_p^c(0)$, which is the value of country reentering the market. Using (58), they can written as a function of B_0 and τ.

Second equation. Using equations (56) and (43), the fund clearing condition becomes:

$$\begin{aligned}
\frac{F}{(1-\alpha)n_0} &= \frac{B_0}{1-\alpha} + \sum_{k=1}^{D-1}\rho^{k-1}\left(\frac{B_0}{\beta^k} + (c-\bar{l}+\tau)\frac{1-\beta^{-k}}{1-\beta}\right), \\
&= B_0\left(\frac{1}{1-\alpha} + \frac{1-\rho^{D-1}\beta^{-(D-1)}}{\beta-\rho}\right) + \frac{c-\bar{l}+\tau}{1-\beta}\left(\frac{1-\rho^{D-1}}{1-\rho} - \frac{1-\rho^{D-1}\beta^{-(D-1)}}{\beta-\rho}\right).
\end{aligned} \tag{61}$$

Third equation. The third equation is the budget constraint of the fund (37):

$$(1-\beta)F = n_p \tau. \tag{62}$$

The three linear equations (60), (61), and (62) form a linear system in three unknowns in B_0, F, τ. Except particular cases (which would correspond to a non-invertible matrix for the linear system – which is a zero-measure set), there is always a solution. These three equations insure that there exist a fund economy equilibrium with a finite F, and $q = \beta$.

F Proof of Proposition 5

In the Hegemon economy, the quasi-planner program can be expressed recursively as follows:

$$\tilde{W} = \max_{\substack{\{\tilde{q}, \tilde{D}, \tilde{c}_p^d, \tilde{l}_p^d, \tilde{c}_u^d, \tilde{c}^H, \tilde{l}^H\}, \\ \{\tilde{l}_{k0}\}_{-1 \le k < \tilde{D}-1}, \{\tilde{c}_k\}_{1 \le k < \tilde{D}-1}, \{\tilde{B}_k\}_{0 \le k < \tilde{D}-1}, \tilde{B}^{H'}}} \tilde{U} + \beta \tilde{W}', \tag{63}$$

subject to the set of country budget constraints (similar to (14)–(16) in the market economy):

$$\tilde{B}_{k-1} = \tilde{c}_k - \bar{l} + \tilde{q}\tilde{B}_k, \quad 1 \le k < \tilde{D} - 1, \tag{64}$$

$$\tilde{B}_{\tilde{D}-2} = \tilde{c}_{D-1} - \bar{l} + \tilde{q}\,(1-\rho)\,\tilde{B}_{\tilde{D}-1} + \tilde{q}\rho\underline{B}, \tag{65}$$

$$\tilde{c}_0 + \tilde{q}\tilde{B}_0 = \tilde{l}_{k0} + \tilde{B}_k, \quad -1 \le k \le \tilde{D} - 1, \tag{66}$$

$$\tilde{c}_p^d = \varphi \tilde{l}_p^d, \text{ and } \tilde{c}_u^d = \varphi \bar{l}, \tag{67}$$

$$\tilde{c}^H = y^H + \tilde{B}^H - \tilde{q}\tilde{B}^{H'}. \tag{68}$$

and subject to participation constraints (similar to (17) and (19) in the market economy): $\tilde{V}_{k0} \ge \tilde{V}_p^d(\underline{B}), \quad -1 \le k < D$ and $\tilde{V}_k \ge \tilde{V}_u^d(\underline{B}), \quad 1 \le k < D$. The planner understands the effect of the saving decisions of the Hegemon and of all other countries on the safe asset price.

$$\sum_{k=0}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k + (1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho\tilde{n}_{\tilde{D}-1}\underline{B} + m\tilde{B}^{H'} = 0. \tag{69}$$

Euler equations can be written as:

$$\tilde{q} + \lambda = \beta\left(\alpha + (1-\alpha)\,u'\left(\tilde{c}_1\right)\right), \tag{70}$$

$$\tilde{q}u'\left(\tilde{c}_k\right) + \lambda = \beta\left(1 - \rho + \rho u'\left(\tilde{c}_{k+1}\right)\right), \quad 1 \le k \le \tilde{D} - 2, \tag{71}$$

$$q\tilde{u'}\left(c_{\tilde{D}-1}\right) + \lambda \ge \beta, \text{ and } \tilde{q} + \lambda = \beta. \tag{72}$$

The price is determined by the following first-order condition:

$$\begin{aligned} 0 = mu'\left(\tilde{c}^H\right)B^{H'} + \sum_{k=-1}^{D-1} n_k^p B_k u'\left(c_k^p\right) + \sum_{k=1}^{D-2} n_k^u B_k^u u'\left(c_k^u\right) \\ + n_{D-1}^u\left((1-\rho)B_{D-1}^u + \rho\underline{B}\right)u'\left(c_{D-1}^u\right). \end{aligned} \tag{73}$$

An equilibrium is $u'(\tilde{c}^H) = u'(c_k^p) = u'(c_k^u) = u'(c_{D-1}^u) = 1$. We obtain: $u'^{-1}(1) = y^H + (1-\tilde{q})\,\tilde{B}^H$ and $\tilde{B}^H = -\frac{y^H - u'^{-1}(1)}{1-\beta}$.

G The Hegemon economy

The asset choice of the Hegemon is derived by the solution of the problem (40)–(41). Even though the function $B \mapsto q(B)$ may be discontinuous, it can be shown to be globally decreasing. Furthermore, we can use the results of Clausen and Strub (2016) (see in particular their section 5.1) to show that the optimum is characterized by the first-order condition $q(B^H)+q'\left(B^{H'}\right)B^{H'} = \beta$, which implies that the equilibrium price is strictly above β.

G.1 Constrained-efficient equilibrium.

The quasi-planner aims at maximizing the instantaneous aggregate utility $\tilde{U}$ expressed as:

$$\tilde{U} = mu\left(\tilde{c}^H\right) + \sum_{k=-1}^{\tilde{D}-1} \tilde{n}_{k0}\left(u\left(\tilde{c}_0\right) - \tilde{l}_{k0}\right) + \sum_{k=1}^{\tilde{D}-1} \tilde{n}_k\left(u\left(\tilde{c}_k\right) - \bar{l}\right) \tag{74}$$
$$+ \tilde{n}_p^d\left(u\left(\tilde{c}_p^d\right) - \tilde{l}_p^d\right) + \tilde{n}_u^d\left(u\left(\tilde{c}_u^d\right) - \bar{l}\right),$$

where the tilde on variables highlights their dependence in $\tilde{D}$. The following proposition characterizes the constrained-efficient equilibrium.

Proposition 5 (Constrained-efficient allocation) *Either the constrained-efficient allocation is autarky or it characterized by:*

- *the asset price is equal to $\tilde{q} = \beta$;*
- *there is full insurance: $\tilde{c}_k = \tilde{c}^H = u'^{-1}(1)$ for $k = 0, \ldots, \tilde{D}-1$;*
- *the net position $\tilde{B}^H$ of the Hegemon is determined by the sign of $u'^{-1}(1) - y^H$. If it is positive and $u'^{-1}(1) \geq y^H$, then the Hegemon is a net lender and if the opposite holds, then the Hegemon is a net borrower.*

The two first items of Proposition 5 are similar to those of Proposition 3 (in absence of Hegemon). The novel part consists in the characterization of the net position of the Hegemon. The budget constraint (41) of the Hegemon coupled with the two first points of Proposition 5 imply that the net borrowing position of the Hegemon is defined by $\tilde{B}^H = \frac{u'^{-1}(1)-y^H}{1-\beta}$, thereby proving the last point. Another take-away of Proposition 5 is that when the Hegemon internalizes the effect of its choices on the asset price, then the market allocation is not constrained-efficient.